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Dynamic characteristics and control of magnetorheological/electrorheological sandwich structures: A state-of-the-art review

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Abstract

During past four decades, applications of magnetorheological and electrorheological fluids in adaptive sandwich structures have been widely studied, primarily for the purpose of vibration control. The rapid response time of controllable magnetorheological/electrorheological fluids to an applied magnetic/electric field and reversible variations in their stiffness and damping properties have been the key motivations for adaptive structures applications. This article presents a comprehensive review of the reported studies on applications of magnetorheological/electrorheological fluids for realizing active and semi-active vibration suppression in sandwich structures. The review focuses on methods of characterizing the magnetorheological/electrorheological fluids in the pre-yield region, magnetic/electric field-dependent phenomenological models describing the storage and loss moduli of fluids, experimental and analytical methods developed for vibration analysis of sandwich structures with magnetorheological/electrorheological fluid treatments, analysis of structures with partial magnetorheological/electrorheological fluid treatments and optimal treatment locations, and developments in control strategies for vibration suppression of magnetorheological/electrorheological sandwich structures. The studies on dynamic responses of fully and partially treated magnetorheological/electrorheological-based sandwich beams, plates, shells, and panels are also discussed, including the mathematical modeling methods and associated assumptions, methods of solutions, and experimental methods.

Keywords

magnetorheological fluid, electrorheological fluid, sandwich structure, vibration and control analysis

Introduction

The merits of sandwich structures with different viscoelastic materials have been well-established for vibration attenuation, especially for their ease of application and low cost (Nashif et al., 1985). Fixed parameter viscoelastic materials, however, would yield limited vibration attenuation performance and may be tuned for isolation within a narrow frequency band. The smart fluids/elastomers with variable stiffness and damping properties, attributed to the changes in their rheology in response to an applied electric/magnetic field, offer attractive potentials for realizing adaptive sandwich structures with enhanced vibration suppression in a wide frequency range. Huang et al. (1996) compared the vibration attenuation of a sandwich plate with passive constrained layer damping (PCLD), active constrained layer damping (ACLD), and active control (AC). The study demonstrated superior performance of the structure with ACLD and AC compared to that with PCLD, although the core layer reduced the actuation ability of actuators employed in the ACLD. Furthermore, the ACLD treatments provided more effective vibration mitigation than the AC under low feedback gains, when a simple derivative (velocity) feedback control law was employed. The AC, however, resulted in better performance, when constraints on the feedback gain and damping layer thickness were relaxed. Nayak et al. (2011) reported 30% greater vibration attenuation of a cantilever sandwich beam containing smart elastomer as the core layer compared

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to that of the structure with a viscoelastic material core. The smart elastomer–based structures, however, exhibit greater regions of instability compared to viscoelastic sandwich structures when subjected to axial loads. It is worth noting that vibration suppression of sandwich structures with piezoelectric materials has also been widely reported (Hosseini-Hashemi et al., 2010a, 2010b, 2012).

Magnetorheological (MR) fluids are smart controllable fluids, which can alter their rheological properties (elasticity, plasticity, and viscosity) from free-flowing condition to semi-solid state rapidly and reversibly in response to an applied magnetic field. MR fluids are a suspension of micron-sized (3–5 μm) ferromagnetic particles of high magnetization saturation in a carrier fluid, generally a type of oil. Coarser particles have also been used, which yield relatively higher dynamic yield stress compared to the finer particles, while the yield stress at saturation of the MR fluid does not depend on the particle size (Genc and Phule, 2002). Magnetorheological elastomers (MREs), also a class of intelligent materials, are composed of magnetic particles (in the 1–100 μm range) suspended in polymer/elastomer or gel-like matrix. The matrix prevents the particles to settle down. MREs are typically fabricated using two different techniques. If the mixture of elastomer matrix and ferromagnetic particles is cured under the presence of applied magnetic field, the formed chain-like structures are locked in their place and thus generate an anisotropic-type MREs. While if during curing process, no external magnetic field is applied, the magnetic particles are randomly dispersed in the matrix, thus generating isotropic MRE. Response time of anisotropic MREs is much less compared with that of MR fluids as the chain-like structures have been already locked in the matrix during curing process. MREs are basically complement MR fluids in the sense that they mainly provide field-dependent modulus while MR fluids provide field-dependent yield stress (Li and Zhang, 2008). Applications of MREs in sandwich beam structures have been widely reported (Choi et al., 2008; Korobko et al., 2012; Nayak et al., 2010, 2012a; Ni et al., 2011; Ying and Ni, 2009; Zhou and Wang, 2005, 2006a, 2006b, 2006c). Such smart materials have also been explored for tunable vibration absorbers to suppress vibration of structures (Deng and Gong, 2007; Gandhi and Thompson, 1990; Ginder et al., 2001; Xu et al., 2010; Zhang and Li, 2009).

Electrorheological (ER) fluids, similar to MR fluids, exhibit changes in rheological properties under a varying electric field. Conventionally, these materials are fabricated by suspending semiconducting solid particles in a dielectric carrier liquid. Although functionality of the MR fluids subjected to magnetic field and the ER fluids subjected to electric field is similar to some extent, the fluids exhibit distinctly different characteristics, which distinguish them with regard to their performance and potential applications. For instance, MR fluids can provide greater changes in their rheological properties and higher yield stress in the presence of magnetic field compared to the ER fluids exposed to an electric field. Weiss et al. (1993) reported that the shear yield stress of MR fluids may change from 2 to 3 kPa in the absence of magnetic field to 100 kPa under magnetic field of 3000 Oe. The ER fluids exhibit substantially lower maximum shear yield stress, in the order of 5 kPa under an electrical field strength of 4 $kV\ mm^{-1}$ (Weiss et al., 1994). Yalcintas and Dai (1999) reported that for the same applied field strength and sandwich beam size, the shifts in natural frequencies were almost two times higher for MR sandwich beam compared to those of the ER sandwich beam. Moreover, sedimentation of the solid particles in ER fluid together with higher sensitivity to impurities and temperature, requirements of relatively higher voltage, and the variations in the material response in electric-time conditions may limit its applications (Yalcintas and Dai, 1998, 1999). Yalcintas and Dai (1999) suggested that MR fluids are well-suited for vibration suppression of structures subject to high-frequency excitations, while ER materials were recommended for vibration suppression under lower frequency excitations. It is worth noting that the ER fluids respond in a similar manner to both alternating current (AC) and direct current (DC) electric fields (Shiang and Coulter, 1996). Applications of ER fluids in sandwich structures and vibration absorbers have also been widely reported. (Ehrgott and Masri, 1992; Gandhi et al., 1988; Kordonsky et al., 1994; Lee, 1992; Park et al., 1994; Rahn and Joshi, 1998; Shiang and Coulter, 1994; Sprecher et al., 1991; Wei et al., 2005).

Vibration analysis and control of sandwich structures necessitate accurate characterization of the ER or MR fluid core layer in the pre-yield region (Weiss et al., 1994). Operating in the post-yield region disrupts the particles suspended in the carrier fluid, which results in sedimentation of the particles, particularly for the ER fluids. Weiss et al. (1994) reported 20%–30% reduction in the storage modulus of the ER fluid under strain level of 1%–10% due to repetitive tests on an adaptive structure. While the ER materials effectively reduce displacement amplitudes of sandwich beams in the linear region, their performance in the non-linear range is limited (Vaičaitis et al., 2008). Experimental characterizations of ER and MR fluids in the pre-yield region have been reported in a number of studies (Claracq et al., 2004; Mohammadi et al., 2010). While the MR/ER fluids were accurately characterized in the pre-yield region, they might be employed fully or partially in the adaptive structures. It is worth noting that partially treated MR- or ER-based sandwich structures may provide superior damping performance compared to the fully treated ones, while having less weight (Rajamohan et al., 2010c; Yalcintas and Coulter, 1998).

The reported studies on sandwich structures employing MR and ER fluids have employed widely different structure analysis methods and experiment methods for characterization of fluids. While the state-of-the-art developments in sandwich structures, ER fluids applications, and MR dampers have been presented in a few review articles (Librescu and Hause, 2000; Stanway et al., 1996; Wang and Liao, 2011), a similar review of developments in vibration analysis and control of MR/ER fluids structures has not yet been presented. A critical review of studies reporting analyses of control of MR and ER fluids sandwich structures including beam, plate, shell, and panel structures is thus presented in this article. The state-of-the-art review focuses on characterization of MR and ER fluids in the pre-yield regime, particularly the frequency- and field-dependent loss and storage moduli; analytical methods and dynamic analyses of fully and partially treated sandwich structures containing controllable ER/MR fluids; and optimization strategies and controller designs to realize vibration suppression corresponding to the selected modes of vibration.

Pre-yield characterization of MR/ER fluids

The properties of MR/ER fluids are strongly related to the applied magnetic/electric field. In the absence of applied field, the suspended particles are randomly dispersed within the carrier fluid. The MR/ER fluid may thus be regarded as a Newtonian fluid, since it exhibits constant viscosity. Moreover, the fluid shows linear relationship between the stress and the strain rate at any point. In the presence of magnetic/electric field, the suspended particles align themselves in the direction of applied field and restrict the motion of MR/ER fluid. The net effect is development of yield stress and apparent viscosity of the fluid. The MR/ER fluid in the presence of magnetic/electric field thus may not be regarded as a Newtonian fluid. In this case, shear stress–shear strain properties of the fluid may be investigated in two regions, referred to as pre-yield and post-yield regions. In the pre-yield region, MR/ER fluid behaves viscoelastically and shear stress and shear strain are proportional in terms of the complex modulus G^* given by (Li et al., 1999)

$$G^* = G' + iG'' \tag{1}$$

where G' is the storage modulus, which determines average energy stored per unit volume of the material over a deformation cycle, and G'' is the loss modulus, which is defined as dissipated energy per unit volume of material in a deformation cycle (Rajamohan et al., 2010c).

The adaptive structures containing MR/ER fluids tend to work in the pre-yield region (Weiss et al., 1994). While the shear strain amplitude experienced by the MR/ER fluids is considered as an important factor which yields the fluids to operate in the pre- or post-yield region, the applied field strength can also change their operation regions. In fact, intensifying the applied field may cause the MR/ER fluids or elastomers to operate in the post-yield region (Hu et al., 2011).

Hu et al. (2011) reported that in the low range of strain amplitude (less than 0.5%, which is near the yield strain of typical MR fluids), the storage modulus of a typical MRE is nearly constant by increasing the strain rate, and the MRE is in the linear (pre-yield) region. This was particularly more evident for magnetic flux density up to 110 mT. Increasing the magnetic flux density, however, caused the MRE to work in the nonlinear post-yield region, even in very low range of the shear strain amplitude, as depicted in Figure 1.

Viscoelastic models representing rheological properties of MR/ER fluids in the pre-yield region

Several models have been developed to identify the storage and loss moduli of the MR/ER fluids in terms of applied field and frequency. Depending on the characteristics of these smart fluids, different phenomenological constitutive models have been employed to describe their rheological behavior in the pre-yield regime. It should be noted that, so far, no comprehensive model has been developed to describe pre-yield behavior of these smart fluids (Mohammadi et al., 2010). That is, depending on the frequency range of interest, applied external field, and properties of the smart fluid, different models should be employed to describe fluids' behavior. Viscoelastic models are the most common models to account for rheological properties of the smart fluids in the pre-yield regime (Gandhi and Bullough, 2005). These models may be described as a combination of springs and viscous dashpots. Viscoelastic models are

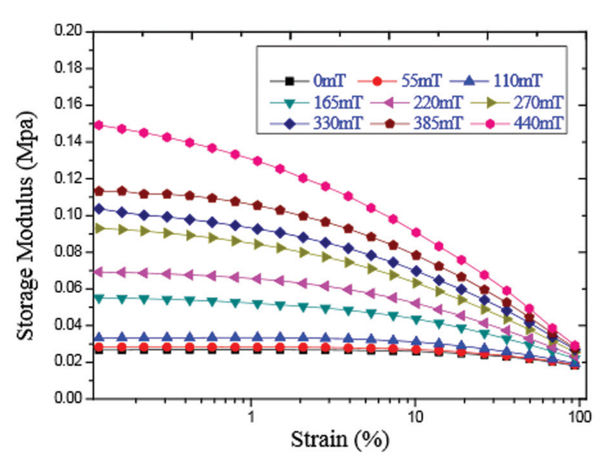
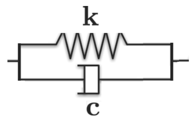
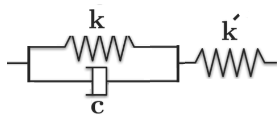
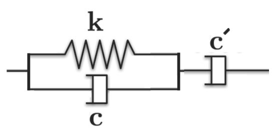


Figure 1. Variations in the storage modulus of MRE with shear strain amplitude, under different levels of magnetic flux density (Hu et al., 2011).

Table 1. Viscoelastic models employed for characterization of MR and ER fluids.

Model	Configuration	Storage modulus	Loss modulus
Solid models			
Kelvin–Voigt solid		$G' = k$	$G'' = c\omega$
Three-parameter solid		$G' = \frac{k'\left[(k+k')k + (c\omega)^2\right]}{(k+k')^2 + (c\omega)^2}$	$G'' = \frac{k'^2 c\omega}{(k+k')^2 + (c\omega)^2}$
Fluid models			
Maxwell fluid		$G' = \frac{k(c\omega)^2}{k^2 + (c\omega)^2}$	$G'' = \frac{k^2 c\omega}{k^2 + (c\omega)^2}$
Three-parameter fluid		$G' = \frac{k(c'\omega)^2}{k^2 + ((c+c')\omega)^2}$	$G'' = \frac{\left(k^2 + (c+c')c\omega^2\right)c'\omega}{k^2 + ((c+c')\omega)^2}$

categorized into two main groups, that is, solid-like models and fluid-like models.

The most common solid-like models representing the pre-yield behavior of the MR/ER fluids are Kelvin–Voigt solid (Mohammadi et al., 2010; Sapiński et al., 2010; Yen and Achorn, 1991) and three-parameter solid (Zener element) models (Gamota and Filisko, 1991), while those of the fluid-like models are Maxwell fluid (Sims et al., 2004) and three-parameter fluid models (Kamath and Wereley, 1997), as illustrated in Table 1. Recently, Li et al. (2010) developed a four-parameter model to illustrate viscoelastic properties of MREs under harmonic loading. The storage and loss moduli of the model, which comprised three-parameter solid model in parallel with a spring (k''), were expressed as

$$G' = \frac{(k'k'' + kk'' + kk')\left[(k+k')^2 + c^2\omega^2\right] + c^2\omega^2 k'^2}{(k+k')\left[(k+k')^2 + c^2\omega^2\right]}$$

$$G'' = \frac{c\omega k'^2}{\left[(k+k')^2 + c^2\omega^2\right]} \qquad (2)$$

In order to determine whether the solid or fluid models can identify pre-yield characteristics of a typical MR or ER fluid accurately, the experimental behavior of the fluid in terms of energy dissipation in a cycle of deformation (Yen and Achorn, 1991), stress–strain hysteresis data (Tang and Conrad, 1996), stress–strain response under constant strain rate amplitude (Sprecher et al.,1987), or variations in the storage and loss moduli with excitation frequency (Claracq et al., 2004) can be monitored and compared with those of the developed models. Yen and Achorn (1991) investigated the loss and storage moduli of an ER fluid in the pre-yield regime, in the range of 1–100 Hz. Their experiment reported modest change in the storage modulus with frequency, which suggests solid-type behavior of the ER fluid. In this case, the Kelvin–Voigt solid model could describe complex shear modulus of the ER fluid, accurately. They also investigated pre- and post-yield behaviors of ER fluid under low- and high-amplitude oscillatory vibrations. Figure 2 illustrates the pre- and post-yield shear stress responses (solid lines) of the ER-based device under oscillatory strain input (dashed line), conducted by Yen and Achorn (1991). It was observed that under low-amplitude strain, the stress–strain has a linear relation. However, increasing the strain amplitude leads the ER fluid into the post-yield region and stress response is non-linear. Furthermore, the shear stress and strain responses are in phase, suggesting negligible damping and predominant elastic behavior of the ER fluid. Yen and Achorn (1991) also observed linear variations in the dissipated energy per cycle of ER device with frequency, which again verified the validity of Kelvin–Voigt solid model to represent pre-yield characteristics of the ER fluid. It should be noted that in the Kelvin–Voigt solid model, the dissipated energy per cycle varies linearly with the applied frequency.

Claracq et al. (2004) employed a rheometer to determine the variations in the loss and storage moduli of a typical MR fluid with frequency. The results suggested

Figure 2. Pre- and post-yield shear stress responses (solid lines) of an ER-based device under oscillatory strain input (dashed line) (Yen and Achorn, 1991): (a) pre-yield region and (b) post-yield region.

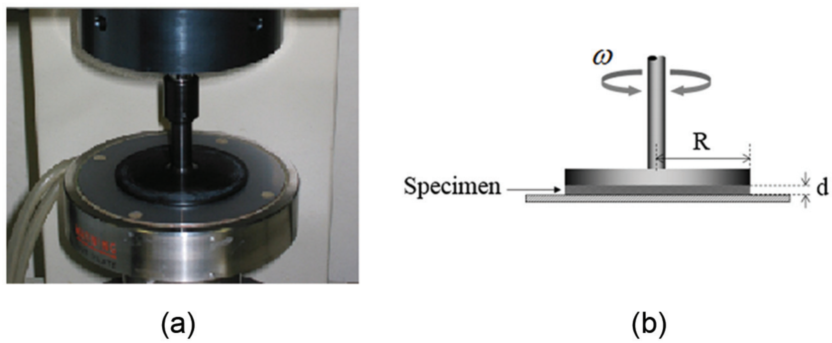


Figure 3. (a) Real image and (b) schematic diagram of a rotational rheometer (Murata, 2012).

insignificant variations in the storage modulus with frequency while the loss modulus varied linearly. This behavior could be represented by Kelvin–Voigt solid model comprising a strong spring in parallel with a weak dashpot. Tang and Conrad (1996) and Sprecher et al. (1987) used the variations in the shear stress with shear strain to identify rheological behavior of MR and ER fluids, respectively, in terms of excitation frequency and applied field. Their findings showed linear variations in the shear stress with shear strain in the pre-yield region. In these studies, the rheological behaviors of the fluids suggested application of the solid models such as three-parameter model to represent the storage and loss moduli. Sims and Wereley (2003) employed the Maxwell fluid model to characterize an ER fluid in the pre-yield regime. In the Maxwell model, the loss and storage moduli approach 0 in small values of the excitation frequency, while in practice the pre-yield region is an elastic region and stiffness never fades out. In order to compensate this error, they employed a bi-viscous element for the damper in the model. This model, however, was not a physical model and represented behavior of the fluid mathematically. Considering different aspects of solid and fluid models, Gandhi and Bullough (2005) suggested that the solid models are more appropriate to identify pre-yield characteristics of the MR/ER fluids.

Methods of characterizing MR/ER fluids in the pre-yield region

The MR/ER fluid characterization methods reported in the literature are generally based on four primary methods, namely, the oscillatory shear strain (Mohammadi and Sedaghati, 2012a), rheometry (Mohammadi et al., 2010; Sun et al., 2003), treating MR sandwich beam as a single-degree-of-freedom (SDOF) system (Choi et al., 1990; Rajamohan et al., 2010c), and standardized test method described in ASTM E756-05:2002 (Allahverdizadeh et al., 2013c; Choi et al., 1992; Leng et al., 1997). Review of the studies on characterization of the MR/ER fluids reveals that rheometers have been widely used to characterize the fluids in terms of frequency and applied field. These devices, which work in either oscillatory or rotational mode, comprise two parallel plates and the MR/ER fluids fill the gap between the two plates, as depicted in Figure 3.

In rotational mode, the lower plate is fixed and the upper one rotates and a sensor measures the torque and related external forces (Mohammadi et al., 2010). The fluid experiences a constant shear strain across the gap, while the strain varies with radial displacement. The pre-yield storage and loss moduli of the fluids are measured at the edge of the top plate (Hirunyapruk et al., 2010). In the oscillation mode, an oscillating plate connected to the shaker is placed between two parallel plates containing the MR/ER fluid. This symmetric arrangement prevents unwanted coupling between the in-plane and transverse motion of the central plate (Sun et al., 2003). Dynamic signal analyzer uses the measured central plate displacement and axial forces on the fixed plates to identify the storage and loss moduli of the fluids in terms of frequency and applied field (Mohammadi and Sedaghati, 2012a). The measurements may be performed in two different modes, namely, amplitude sweep mode and frequency sweep mode (Mohammadi et al., 2010). The amplitude sweep test varies the amplitude of strain under constant excitation frequency to identify the maximum shear stress corresponding to the linear behavior of the fluid. Then, frequency sweep test is employed to find the storage and loss moduli of the fluid in terms of frequency. It is worth noting that the frequency sweep test may be conducted in constant shear rate (CSR) or constant shear stress (CSS) modes (Mohammadi et al., 2010).

Application of sandwich structure as SDOF for pre-yield characterization of MR/ER fluids is preferable to the rheometer (Shiang and Coulter, 1996), which is due to smaller strain amplitude of the fluid in the sandwich structure compared to the rheometer. Choi et al. (1990) investigated free vibration of a hollow beam of polystyrene filled by ER fluid to find the complex shear modulus of the fluid. The composite beam was considered as a viscoelastic element and modeled as an SDOF system. Considering small thickness of polystyrene employed to fabricate hollow beam, the shear modulus of the structure was considered to be equal to that of the ER fluid. Employing free vibration analysis and obtaining the natural frequencies and logarithmic decrement of the structure, the loss and storage moduli of the fluid were obtained. The developed model, however, could not capture frequency-dependent behaviors of the storage and loss moduli. Rajamohan et al. (2010c) employed the same procedure on MR sandwich beam of aluminum face layers to characterize the fluid, but the effect of face layers was not taken into account. They designed an optimization problem to update the loss and storage moduli obtained from the experiment to achieve better agreement with the experimental results, in terms of resonant frequencies.

ASTM E756-05:2002 (2002) is a standard test for characterizing viscoelastic materials in the linear region. The standard employs cantilever sandwich beam to predict rheology of polymer materials sandwiched in the core layer. Choi et al. (1992) employed this standard to identify rheological behavior of the ER fluid. They outlined poor accuracy of the method for demonstrating rheological behavior of the sandwich beam structures with ER fluid. Their study suggested a decrease in the derived moduli of the composite beam with an increase in the applied field. They related this anomalous behavior to deviation of beam deformation from the assumptions considered in the American Society for Testing and Materials (ASTM) equations. Allahverdizadeh et al. (2013a, 2013c) also employed this technique to characterize ER fluid in the pre-yield region. They reported that the standardized ASTM E756-05:2002 method provides a rough estimation of the storage and loss moduli of the fluid and is not accurate, which is in part attributed to neglecting contribution due to the sealant. Furthermore, this method cannot capture frequency-dependent behavior of the fluid and is more applicable for sandwich structures with solid viscoelastic materials as the core layer. Allahverdizadeh et al. (2013a, 2013b, 2013c) suggested that the ASTM method should be accompanied by an optimization process to update and modify the extracted data. Consequently, they employed particle swarm optimization (PSO) technique to seek optimal storage modulus of the ER fluid by matching the resonant frequencies obtained by theory and experiment. Then, the optimum loss modulus was obtained by matching the theoretical resonance amplitudes with those of the experiment.

Mathematical representation of the loss and storage moduli

It has been widely reported that the storage (G') and loss moduli (G'') of the MR/ER fluids, prior to saturation, can be described by quadratic functions in the magnetic flux density/electric field (B/E) (Allahverdizadeh et al., 2013a; Choi et al., 1990; Mohammadi et al., 2010; Rajamohan et al., 2010a, 2010b, 2010c). This is attributed to rheological behaviors of the MR/ER fluids which depend on the dipole–dipole interactions. These interactions are proportional to the product of B/E and the dipole moment, P. The dipole moment is also proportional to B/E prior to the saturation of the MR/ER fluids; consequently, the rheological properties of the MR/ER fluids such as loss and storage moduli and yield stress are quadratic functions of B/E (Choi et al., 1990). Mohammadi et al. (2010) identified rheological properties of two smart fluids including a ferromagnetic nano-particle fluid and an MR fluid using rheometer. In the frequency domain, Kelvin–Voigt solid and the three-parameter fluid models were employed to represent the pre-yield behaviors of the ferromagnetic nano-particle and MR fluids, respectively. They also suggested quadratic polynomials to represent storage modulus of the fluids in terms of magnetic field strength. This model was developed for a limited range of frequency in which the storage modulus no longer depends on the excitation frequency. Since no value of magnetic field was found for which the loss modulus was independent of frequency, they could not present explicit functions representing the loss factors in terms of magnetic field density.

Ginder et al. (1995) reported that although at very low levels of applied magnetic field, the rheological properties of the MR fluid may be represented by a quadratic function in terms of magnetic field strength, at flux density above linear region but lower than what is needed for complete saturation of the MR fluid, the rheological properties of the fluid are proportional to $B^{3/2}$. The properties of the magnetically saturated MR fluid, however, do not depend on the magnetic flux

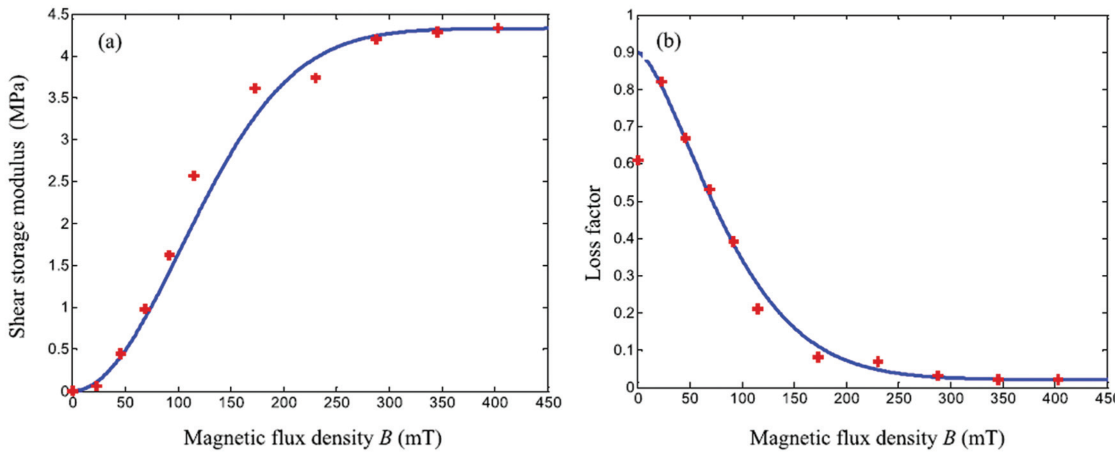


Figure 4. Variations in the (a) shear storage modulus and (b) loss factor of MRF 140CG with magnetic flux density. Measurement (+) and empirical models (−) (Hirunyapruk et al., 2010).

density. Yalcintas and Dai (2004) and Mikhasev et al. (2011) proposed linear functions in terms of magnetic flux density to represent complex shear modulus of the MR fluids. Yanju et al. (2001) suggested Hyperbl functions $((aE)/(b+E))$ to represent the storage and loss moduli of a typical ER fluid, where a and b were the parameters related to the material and excitation frequency and E represented the electric field strength. Hirunyapruk et al. (2010) proposed two exponential functions to identify the storage modulus and loss factor of a typical MR fluid (MRF 140CG) in the pre-yield region. The frequency-independent equations, which predicted the storage modulus and loss factor up to magnetic saturation, were expressed as

$$\begin{aligned} G' &= G'_z + (G'_\infty - G'_z)(1 - e^{-\alpha_1 B^{\alpha_2}}) \\ \eta &= \eta_\infty + (\eta_z - \eta_\infty)e^{-\alpha_3 B^{\alpha_4}} \end{aligned} \quad (3)$$

where G' and η represent the storage modulus and loss factor, respectively. $\alpha_1, \alpha_2, \alpha_3, \alpha_4, G'_z, G'_\infty, \eta_z, \eta_\infty$ are the empirical constants identified by fitting the models to the experimental data. Figure 4 illustrates the variations in the storage modulus and loss factor of the fluid with the magnetic flux density (Hirunyapruk et al., 2010). The experimental results obtained by rheometer were shown by bullet points while those obtained by the developed model were depicted by solid lines. The results suggest saturation of the fluid around 250 mT. Furthermore, the storage modulus and loss factor indicate quadratic variations with the magnetic flux density, in the pre-saturation region.

Equivalent linearized complex moduli of the ER/MR fluids may be employed to represent the fluids behavior in the post-yield region (Allahverdizadeh et al., 2014; Lee, 1995; Lee and Cheng, 1998). The experiments conducted by Stevens et al. (1987) suggested a constitutive relationship for the ER fluid under quasi-static shearing as

$$\tau = G_0\gamma(1 - e^{-(\gamma_0/\gamma)}) + \mu\dot{\gamma} \quad (4)$$

where τ and γ denote shear stress and shear strain, respectively. G_0 represents linear shear modulus of infinitesimal shear strain. γ_0, which is a model parameter, and G_0 are the electric field-dependent functions and normally represented in a quadratic form (Allahverdizadeh et al., 2013a, 2014; Lee and Cheng, 2000). It should be noted that the shear stress due to viscosity of the fluid, μ, was much lower than that of the exponential function; thus, the effect of viscosity neglected from the equation (Allahverdizadeh et al., 2013a; Lee, 1995). In order to obtain the pre-yield equivalent moduli at different amplitudes of strain, the hysteresis loop of ER fluid under sinusoidal strain excitation was constructed. The linear equivalent storage and loss moduli of the ER fluid in cyclic loading were obtained to yield the same strain energy and dissipated energy of the fluid. Having found the equivalent pre-yield storage and loss moduli of the fluid, the viscoelastic models could be employed to investigate dynamic responses of ER/MR sandwich structures (Allahverdizadeh et al., 2013a; Lee, 1995). It is worth noting that the pre-yield behavior of MR/ER fluids may be predicted using post-yield characterization of these fluids. Choi et al. (1992) and Mohammadi and Sedaghati (2012a) extrapolated the post-yield characteristics of ER fluid to identify pre-yield properties. Genc and Phule (2002) conducted the same analysis for the MR fluids.

Dynamic characteristics of fully treated MR/ER sandwich beam structures

Fabrication and experimental study of MR/ER sandwich beam structures

The concept of sandwich structures containing ER and MR fluids as the core layer was introduced in the patents issued by Carlson et al. (1990) and Weiss et al.

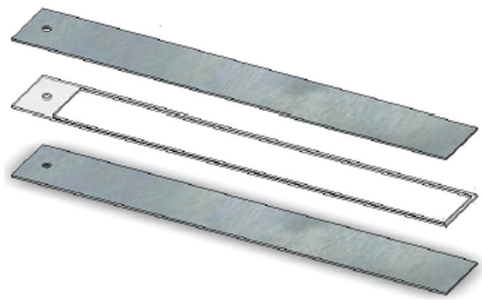


Figure 5. Sketch of a sandwich beam structure (Lara-Prieto et al., 2010).

(1996), respectively. The primary studies on dynamic characteristics of MR/ER sandwich structures were conducted on the multi-layer sandwich beam, which was due to its relatively simple mechanical model (Lara-Prieto et al., 2010). Furthermore, the experimental studies on MR/ER sandwich beam structures have been mostly conducted on the cantilever sandwich beams (Berg et al., 1996; Chen et al., 1994; Don and Coulter, 1995; Gandhi et al., 1989; Phani and Venkatraman, 2003, 2005; Rajamohan et al., 2010a, 2010b, 2010c; Wei et al., 2011), although some studies on clamped–clamped (Haiqing and King, 1997) and simply supported structures (Lee and Cheng, 1998; Sun et al., 2003; Yalcintas and Coulter, 1995a; Yalcintas et al., 1995) have also been reported. The MR/ER sandwich beam, as depicted in Figure 5, consists of two elastic layers, MR/ER fluid core layer and spacer, which provides a gap between face layers and prevents fluid from leakage. It is worth noting that Qiu et al. (1999) conducted an experiment on five-layer sandwich structure comprising three elastic and two ER core layers to increase the effect of fluid in vibration suppression.

Face layers. Review of literature shows that the elastic layers employed in the MR sandwich structures were typically chosen from aluminum strips (Bishay et al., 2010; Hu et al., 2011; Joshi, 2012; Lara-Prieto et al., 2010; Rajamohan et al., 2010a, 2010b, 2010c; Sapiński and Snamina, 2009; Sun et al., 2003; Yalcintas and Dai, 1999, 2004; Yeh and Shih, 2006a; Zhou et al., 2006). It is attributed to low damping properties and relatively high stiffness of aluminum compared to that of the MR fluid. Furthermore, relative magnetic permeability of aluminum is equal to 1, which ensures uniform distribution of the magnetic field applied to the structure (Sun et al., 2003). The face layer materials, however, have no effect on uniformity of the electric field applied to the core layer of ER sandwich structures; hence, different materials such as aluminum (Choi et al., 1992; Don and Coulter, 1995; Shiang and Coulter, 1996; Wei et al., 2011; Yalcintas and Coulter, 1995b), steel (Berg et al., 1996; Haiqing and King, 1997; Phani and Venkatraman, 2003), polystyrene (Choi et al., 1992), and functionally graded material (FGM) (Allahverdizadeh et al., 2013c) have been employed to fabricate the ER-based sandwich structures. MR sandwich beams could also employ some other materials such as conductive (Choi et al., 2010; Nayak et al., 2010) and polyethylene (PET)-based materials (Hirunyapruk et al., 2010; Lara-Prieto et al., 2010) as the face layers. Hirunyapruk et al. (2010) and Lara-Prieto et al. (2010) utilized non-metallic materials of perspex and PET as the elastic layers, respectively. These materials were transparent and ensured that no bubbles were left within the fluid during the fabrication process (Lara-Prieto et al., 2010). Joshi (2012) fabricated an adaptive structure consisting of a cantilever hollow pipe of stainless steel which encompassed inner wooden layer and an embedded layer of MR fluid. Due to rigidity of the structure, the MR fluid was not subjected to significant shear deformation; hence, no noticeable variations in the dynamic responses of the structure were reported, in response to the applied magnetic field.

Choi et al. (2010) employed steel skins and MRE as the core layer to fabricate an adaptive sandwich beam. They investigated the effect of steel skin face layers on disturbing homogeneity of the magnetic flux applied to the structure. The results illustrate that the sandwich beam with thick steel skins induces higher value of magnetic field than that with thin steel face layer, under the same external conditions. Furthermore, under dynamic deformation of conductive skins, motion-induced eddy current is generated on the face layers so that the magnetic field closed to the face layers is disturbed and magneto-elastic load is applied to the face layers (Zhou and Wang, 2006c). The magneto-elastic load consists of Lorenz body force and the surface force caused by the Maxwell's stress applied on the surface of the conductive skins. However, the results suggest insignificant effect of magneto-elastic loads on the dynamic properties of the sandwich beam. It should be noted that the bulk magnetic permeability of the MR fluids is in the range of 3–9, which is significantly lower than that of steel (around 500). Therefore, the magneto-elastic force applied to the MR fluid has no significant effect on the vibration analysis of the MR-based sandwich structures (Zhou and Wang, 2006c).

Sealant and spacer. In order to maintain uniform gap between face layers and contain the fluid as the core layer, spacer and sealant are required to adhere around the edges. For this purpose, applications of plastic spacer (Sun et al., 2003; Wei et al., 2008), silicon rubber (Allahverdizadeh et al., 2013c, 2014; Berg et al., 1996; Choi et al., 1992; Yalcintas and Coulter, 1995a), latex materials (Don and Coulter, 1995; Shiang and Coulter, 1996), polycarbonate (Don and Coulter, 1995; Shiang and Coulter, 1996; Yalcintas et al., 1995), perspex (Phani and Venkatraman, 2003, 2005), and Buna-N rubber (Rajamohan et al., 2010a, 2010b, 2010c) have been widely reported in the literature, which is due to flexibility and oil resistance properties of these materials. Bishay et al. (2010) employed aluminum frame to provide a uniform gap between the face layers of MR-based sandwich beam. The high flexural rigidity of the aluminum frame prevented the structure to experience significant shear deformation in the core layer; hence, the effect of MR fluid on the dynamic characteristics of the structure was insignificant, in response to applied magnetic field. Application of PET frame and tape for sealing the MR fluid have also been reported by Lara-Prieto et al. (2010) and Hirunyapruk et al. (2010), respectively. Some studies have considered the effect of the sealant and spacer in the mathematical modeling (Allahverdizadeh et al., 2013c, 2014; Bishay et al., 2010; Kang et al., 2001; Lee, 1995; Rajamohan et al., 2010a, 2010b, 2010c). They employed rule-of-mixture to account for the effect of silicon rubber on the complex shear modulus of the core layer. Based on rule-of-mixture, the homogenized complex shear modulus of the middle layer can be expressed as

$$\bar{G} = G_r\left(\frac{b_r}{b}\right) + G^*\left(1 - \frac{b_r}{b}\right) \tag{5}$$

where G_r and G^* are the shear moduli of the rubber and fluid, respectively, while b_r and b are the associated widths of the rubber and sandwich beam, respectively. The material properties of the rubber may be provided by experiment (Lee and Cheng, 1998) or supplier (Rajamohan et al., 2010c).

Applying magnetic/electric field over MR/ER sandwich structures. While application of magnetic field over MR sandwich structure was accompanied with some complexities which limited maximum applied field and coverage area, the electric field over ER sandwich structures could be provided easily. The face layers of ER sandwich structures served as the electrodes for the applied electric field through high-voltage power supply (Choi et al., 1993, 1994; Wei et al., 2007, 2011). The maximum electric field strength reported in the literature is in order of 4 kV mm^{-1}. A thorough review of the reported studies shows that the permanent magnets were widely used to generate magnetic flux over the MR sandwich structures. Different intensities of the magnetic field were realized by varying the vertical position of the permanent magnets with respect to the sandwich structure. The vast majority of the reported studies using permanent magnets with MR sandwich beam structures have considered substantially low field density, well below the magnetic saturation of the fluids (around 700 mT). Rajamohan et al. (2010a, 2010b, 2010c) characterized the fluid properties and evaluated responses of an MR sandwich beam under the field up to 50 mT. Joshi (2012), Yalcintas and Dai (1999, 2004), Sun et al. (2003), Bishay et al. (2010), and Hu et al. (2011) conducted similar studies with fields up to 55, 70, 90, 100, and 100 mT, respectively. It is very difficult to achieve a uniform field density of higher magnitude with permanent magnets, partly due to limited clearance between the structure and the magnets, although it would be possible to realize a stronger field locally at some points on the structure. Choi et al. (2010) and Lara-Prieto et al. (2010) used somewhat higher field density in the order of 300 and 320 mT, respectively. The magnetic flux distributions over the structures in these two studies were non-homogenous, which was attributed to the usage of several magnets to generate the magnetic flux. In these studies, the magnets were located inside aluminum housings at top and bottom of the MR beam. Since the magnets located in each housing had the same polarity, they repelled each other resulting in gaps between the magnets and thereby non-homogenous magnetic field. Hirunyapruk et al. (2010) employed electromagnet to generate a local magnetic flux of 205 mT over small portion of a tunable MR-filled beam-like vibration absorber.

Experiment methods on MR/ER sandwich beam structures. Three main experiments have been conducted to characterize dynamic responses of the MR/ER multi-layer beam structures in terms of natural frequency and damping properties: free vibration (Choi et al., 1990; Gandhi et al., 1989; Joshi, 2012; Lara-Prieto et al., 2010; Leng et al., 1997; Rajamohan et al., 2010c), impact hammer (Lara-Prieto et al., 2010; Lu and Li, 2007), and shaker excitation (Allahverdizadeh et al., 2013c, 2014; Bishay et al., 2010; Choi et al., 2010; Hirunyapruk et al., 2010; Joshi, 2012; Rajamohan et al., 2010a, 2010b, 2010c; Sun et al., 2003; Zhou and Li, 2003). Lara-Prieto et al. (2010) employed three mentioned methods to investigated dynamic responses of MR sandwich beam. Their results suggest that although the applied force in each case was different and the structure vibrated at different amplitudes, the acquired natural frequencies of the structure were almost the same. Furthermore, pure forces could not be applied to the structure without any interaction between the exciter and structure. In other words, the mass and stiffness effects of the hammer tip and shaker attachment caused some discrepancies between the theoretical and experimental results. The acceleration responses of the MR/ER sandwich structures were measured by accelerometers mounted on the face layers. Although single-axis accelerometers were widely used to measure vibration responses of the structures (Allahverdizadeh et al., 2013c, 2014; Rajamohan et al., 2010a, 2010b, 2010c), application of laser sensors (Choi et al., 2010) and eddy current probe (Lee and Cheng,

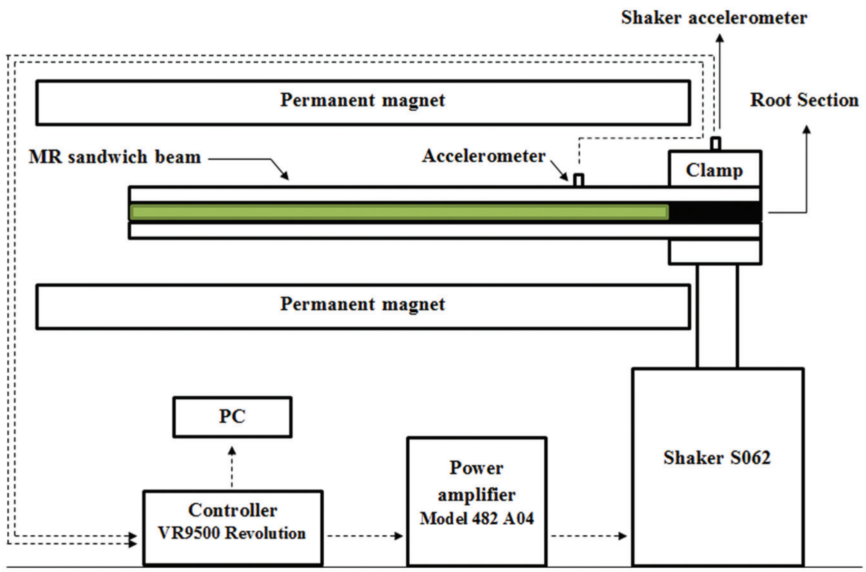


Figure 6. Schematic experimental setup conducted based on vibration excitation of MR sandwich beam (Eshaghi et al., 2015a).

1998; Wei et al., 2007, 2011) for measuring vibration displacement and laser vibrometer (Lara-Prieto et al., 2010) for measuring velocity have also been reported. The measured signals were analyzed in the signal analyzer and the natural frequencies and damping properties of the sandwich beam structures were subsequently identified from the peaks in the frequency response functions. Figure 6 shows schematic of a typical experimental setup representing shaker excitation of a cantilever MR sandwich beam.

Mathematical modeling of MR/ER sandwich beam structures

In the view of viscoelastic behavior of MR/ER fluids in the pre-yield region, all the models demonstrating vibration characteristics of viscoelastic sandwich structures are also potentially applicable to MR/ER adaptive structures. The first model, demonstrating dynamic responses of viscoelastic sandwich structures, was proposed by Ross et al. (1959). This model, which is known as Ross-Kerwin-Ungar (RKU), is based on a modified Euler–Bernoulli beam equation and was expressed as

$$m(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = 0 \tag{6}$$

where $m(x)$ is the mass per unit length and EI denotes the flexural rigidity of the structure, which is presented in terms of material properties and geometry of the viscoelastic sandwich beam. DiTaranto (1965) presented a sixth-order differential equation to characterize governing equations of motion of a three-layer viscoelastic beam structure. Subsequently, Mead and Markus (1969, 1970) modified the DiTaranto model and took the effects of beam transverse inertia into account. Furthermore, in contrast to DiTaranto model, which was applicable to simply supported boundary (SSB) condition under special class of forced vibration, the Mead and Markus (MM) model could investigate the sandwich structures under different geometry boundary conditions. The MM model can be expressed as

$$\begin{aligned}&\frac{\partial^6 w}{\partial x^6} - g(1+Y)\frac{\partial^4 w}{\partial x^4} + \frac{m(x)}{EI}\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right)\\ &= \frac{1}{EI}\left(\frac{\partial^2 q(x,t)}{\partial x^2} - gq(x,t)\right)\end{aligned} \tag{7}$$

where $m(x)$ is the mass per unit length, and g, Y, and EI are defined based on geometry and material properties of the sandwich beam. $q(x, t)$ also represents the harmonic loading over the sandwich structure.

Coulter et al. (1989) and Coulter and Duclos (1990) employed RKU model to investigate dynamic responses of ER sandwich beam structures. Application of DiTaranto and MM models for characterizing vibration behavior of the MR/ER-based sandwich beams has also been widely reported (Hu et al., 2006; Yalcintas and Coulter, 1995a, 1995c, 1998; Yalcintas and Dai, 1999; Yeh and Shih, 2005). Mahjoob et al. (1993) provided a comparison between the results obtained by RKU and MM models and those of the experiment on an ER-based sandwich beam. They suggested more realistic dynamic behavior of the structure predicted by MM model. It is worth noting that since these models have been developed for viscoelastic materials, they might show some inaccuracies in the analysis of MR/ER fluid structures. For instance, Coulter and Duclos (1990) conducted an experiment on ER sandwich structure and realized that application of RKU underestimates both modal frequencies and damping of the structure. They attributed this discrepancy to deviation between the theoretical assumptions and the experimental model. Don and Coulter (1995) suggested that the RKU and MM models can adequately predict dynamic behavior of ER-based adaptive structure if the core layer thickness is uniform and strain in the sandwiched layer is uninhibited.

Finite element (FE) method is the most reported approach in vibration analysis of the MR/ER sandwich structures (Allahverdizadeh et al., 2012, 2013a, 2013c, 2014; Bishay et al., 2010; Hirunyapruk et al., 2010; Lee, 1995; Mohanty, 2013; Nayak et al., 2012b; Rajamohan et al., 2010a, 2010b, 2010c, 2013; Rezaeepazhand and Pahlavan, 2008a; Wei et al., 2011; Zhou et al., 2006). This method employs potential and kinetic energy of the structures and represents the governing equations of motion in a matrix form as follows

$$[m]\{\ddot{x}\} + [k]\{x\} = \{f\} \tag{8}$$

where $[m]$, $[k]$, and $\{f\}$ are the mass matrix, stiffness matrix, and force vector, respectively. Furthermore, application of Hamilton energy method (Chen and Hansen, 2005; Choi et al., 2010; Dwivedy and Srinivas, 2011; Rezaeepazhand and Pahlavan, 2008a; Sun et al., 2003; Yeh et al., 2004) and Ritz method (Rajamohan et al., 2010a, 2010b, 2010c; Rajamohan and Ramamoorthy, 2012) has been widely reported. Sun et al. (2003) employed Hamilton principle to derive governing equations of motion of MR-based sandwich beams, as follows

$$\rho\frac{\partial^2 w}{\partial t^2} + 2E_f I_f\frac{\partial^4 w}{\partial x^4} - G^* bh_2\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \varphi}{\partial x}\right) = f(x,t) \tag{9}$$

where ρ, E_f, I_f, and G^* denote the density of the beam, Young's modulus of each surface layer, moment of inertia at the centroid of elastic layer, and complex shear modulus of the core layer, respectively. Furthermore, b and h_2 are the beam width and core layer thickness, respectively, and $f(x,t)$ and $\varphi(x,t)$ represent the external force applied to the structure and cross-sectional rotation, respectively. The Ritz method employs admissible basis functions to describe displacement of the structure, which satisfy the geometry boundary conditions. For example, the radius-dependent displacement amplitude functions in radial (u), circumferential (v), and transverse (w) directions of an annular circular plate can be described by polynomial functions multiplied by the boundary functions, such that

$$\begin{aligned} u(r) &= F_1(r)H_1(r)\sum_{k=0}^{N} b_k r^k \\ v(r) &= F_2(r)H_2(r)\sum_{k=0}^{N} c_k r^k \\ w(r) &= F_3(r)H_3(r)\sum_{k=0}^{N} l_k r^k \end{aligned} \tag{10}$$

where b_k, c_k, and l_k are the unknown coefficients of the polynomial functions and $F_i(r)$ and $H_i(r)$ are the boundary functions describing the inner and outer edges of the annular plate, respectively. The boundary functions are defined according to the given geometry boundary conditions. k is an integer and N denotes the highest degree of polynomials representing the displacement amplitude functions. This method represents governing equations of motion as follows

$$(\boldsymbol{K} - \omega^2\boldsymbol{M})\boldsymbol{C} = 0 \tag{11}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are the stiffness and mass matrices, respectively, and $\boldsymbol{C}$ is the vector of arbitrary coefficients, used to define displacement fields. It is worth noting that all the above-mentioned methods assumed that the normal stress in the core layer was neglected which was due to negligible Young's modulus of the MR fluid compared to the elastic layers. The fluid layer thickness was assumed to be very small compared to its length and the slippage between the elastic and fluid layers was neglected. The shear strain and stress components in the elastic layers were considered to be negligible, and classical plate theory assumptions were applicable which was due to very small thickness of the elastic layers compared to the length of the beam. The transverse displacement through the structure was considered uniform, and the damping due to elastic layers was also assumed to be negligible (Bishay et al., 2010; Rajamohan et al., 2010a, 2010b, 2010c; Sun et al., 2003; Yalcintas and Dai, 1999, 2004). Table 2 summarizes the models and methods employed to represent governing equations of motion of the sandwich beam structures.

Some studies, however, have not implemented aforementioned assumptions and modeled the sandwich structures with less simplifications. Choi et al. (2010) employed higher order sandwich beam theory and assumed that the applied load could change the thickness of the core layer and core layer cross section may not remain planar. Nayak et al. (2012b) presented a vibration analysis on the MR sandwich beam using two different assumptions and compared the results. In the first case, the classical theory was employed and only the potential energy due to shear deformation of the core layer was considered. In the second case, higher order theory was used to derive governing equations of motion of the structure, and in addition to shear deformation, the potential energy due to transverse and axial deformations in the core layer was considered. The results suggested an insignificant increase in the stiffness and damping properties of the structure under second assumptions compared to those of the first assumptions. Furthermore, the response amplitudes in the second case were found to be less than that of the classical plate theory. Allahverdizadeh et al. (2013b) suggested that the fourth natural frequency of an ER sandwich

Table 2. Mathematical models employed to identify dynamic response of MR/ER sandwich beams.

Model	Equation
RKU	$m(x)\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = 0$
MM	$\frac{\partial^6 w}{\partial x^6} - g(1+Y)\frac{\partial^4 w}{\partial x^4} + \frac{m(x)}{EI}\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right) = \frac{1}{EI}\left(\frac{\partial^2 q(x,t)}{\partial x^2} - gq(x,t)\right)$
Finite element	$[m]\{\ddot{x}\} + [k]\{x\} = \{f\}$
Ritz method	$(K - \omega^2 M)C = 0$
Sun et al. (2003)	$\rho\frac{\partial^2 w}{\partial t^2} + 2E_f I_f\frac{\partial^4 w}{\partial x^4} - G^* bh_2\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial}{\partial x}\right) = f(x,t)$

MM: Mead and Markus.

beam, obtained by Timoshenko and Euler–Bernoulli theories, showed 3% and 16% deviation from the experiment, respectively, when the thickness ratio (thickness to length) of the face layers increased up to 0.1.

Observations and findings

Effect of applied field on the natural frequencies of MR/ER sandwich beams. The reported studies on the dynamic responses of MR/ER multi-layer sandwich beams suggest an increase in the natural frequencies of the structures with increasing the magnetic/electric field. This phenomenon, which was reported in several studies (Allahverdizadeh et al., 2012, 2013a, 2013c, 2014; Choi et al., 1989a, 1989b, 1989c; Choi and Park, 1994; Rajamohan et al., 2010a, 2010b, 2010c; Sepehrinour and Nezami, 2012; Yalcintas and Coulter, 1995a, 1995b, 1995c; Yeh and Shih, 2005), can be attributed to an increase in the complex shear modulus of the MR/ER fluids with increasing the applied field. However, few studies (Bishay et al., 2010; Choi et al., 2010; Hu et al., 2011; Lara-Prieto et al., 2010) reported decrease in the natural frequencies of the MR-based sandwich beam structures with an increase in the applied magnetic field. Bishay et al. (2010) believed that a decrease in the natural frequencies was attributed to the damping effect of MR fluid which was higher than its stiffness effect. Choi et al. (2010) reported that a decrease in the natural frequencies was due to the magnetic preload. The magnetic preload increased the flexibility of the sandwich structure and decreased the natural frequencies (Yin et al., 2006). Lara-Prieto et al. (2010) illustrated that shifting to the lower frequencies in response to increasing the applied magnetic field was attributed to non-uniform magnetic field over the structure, which would also lead to non-uniform concentration of the magnetic particles and stiffening effect of the MR fluid along the sandwich beam. However, the last justification seems more realistic, since no study has reported a decrease in the natural frequencies of fully treated ER sandwich beam in response to increasing the electric field which might be attributed to uniform electric field applied to the ER-based structures.

The variations in the natural frequencies of ER sandwich structures subjected to an electric field are less significant than those of the MR-based structures in response to the magnetic field (Yalcintas and Dai, 1999). Some studies reported insignificant variations in the resonant frequencies of ER sandwich structures in response to the electric field (Rezaeepazhand and Pahlavan, 2008a; Wei et al., 2007). Based on the study conducted by Phani and Venkatraman (2005), although ER fluid filled beam with starch particle concentration of 30% showed no significant variations in the resonant frequencies, in response to the applied electric field, a linear relationship between these two items could be observed in an ER-based sandwich beam with 40% starch particle concentration. It is widely reported that the variations in the resonant frequencies of MR/ER-based sandwich beam structures with applied magnetic/electric field are almost linear (Allahverdizadeh et al., 2013a; Choi et al., 1992; Coulter and Duclos, 1990; Rajamohan et al., 2011). Don and Coulter (1995) announced that although the experimental results indicated linear variations in the resonant frequencies of ER sandwich beam with respect to the applied electric field, the theoretical results showed a parabolic relationship. They believed that linear resonance–electric relationship reported in the experiment was due to overfilling of the core layer. Lu and Li (2007) proposed an exponential function to relate the resonant frequencies of ER-based sandwich beam to the applied electric field as

$$rf_i = rf_o + Ae^{E/t} \quad (12)$$

where $rf_i = f_i/f_{io}$ represents the ratio of ith natural frequency of the sandwich beam under electric field of E to that of the structure with no electric field, f_{io}. rf_o, A, and t are the regression coefficients.

Effect of applied field on the loss factors and deflection of MR/ER sandwich beams. Damping properties of adaptive

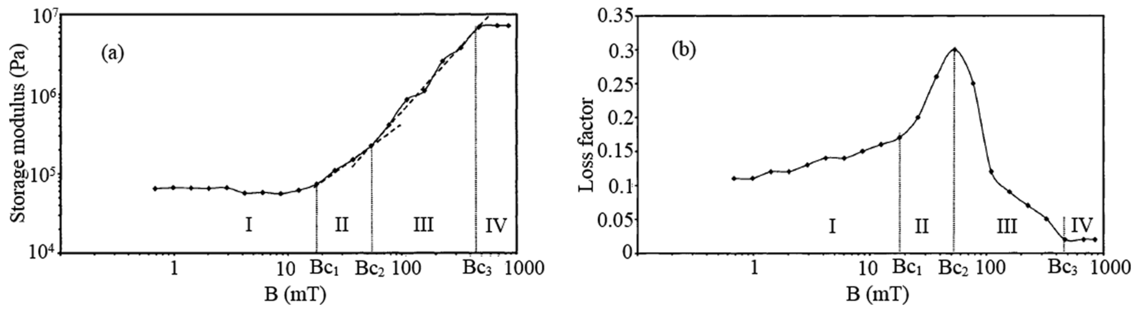


Figure 7. Variations in the (a) storage modulus (Pa) and (b) loss factor of a typical MR fluid with magnetic flux density (Li et al., 2005).

sandwich structures have been widely studied in terms of loss factor (Bishay et al., 2010; Choi et al., 2010; Nayak et al., 2010, 2012a, 2012b; Rajamohan et al., 2010a, 2010b, 2010c, 2013; Sun et al., 2003; Yalcintas and Dai, 1999, 2004; Yeh and Shih, 2006a, 2006b). The loss factor is defined as the ratio of imaginary to real component of complex eigenvalues (Rajamohan et al., 2010c; Sun et al., 2003). The loss factor relates to the dissipated energy as proportion of the stored energy of a material per radian. Although qualitative trend of variations in the natural frequencies of MR/ER sandwich structures in response to the magnetic/electric field is predictable, the structural loss factors exhibit different behaviors. The variations in the loss factors of MR/ER sandwich beam structures under magnetic/electric field depend on both the loss and storage moduli of the fluids. Depending on the behaviors of these moduli, the structural loss factor may be a subject of decrease (Haiqing et al., 1993; Yalcintas and Coulter, 1995b, 1995c) or increase (Choi et al., 1992; Wei et al., 2007; Yalcintas and Coulter, 1995c) with increasing the applied field. In addition to the fluid moduli, the excitation frequency and beam geometry significantly contribute to the variations in the loss factor of the structures under applied field. Lara-Prieto et al. (2010) reported 41% increase in the damping ratio of an MR-based sandwich beam with PET face layers, while the applied magnetic field reached up to 110 mT. The study conducted by Joshi (2012) suggested 59% increase in the damping ratio of an MR multi-layer beam structure with aluminum elastic layers while the magnetic flux density increased from 30 to 55 mT. The more significant increase in damping of aluminum sandwich structure compared to PET one might be attributed to negligible damping of aluminum face layers, while in PET sandwich beam the face layers contributed to the damping properties of the structure, significantly. Therefore, application of magnetic field caused more significant variations in the damping of aluminum sandwich beam compared to PET one.

It is widely reported that the loss factors corresponding to the lower modes of MR/ER-based sandwich structures increase with increasing the applied field and this trend is reversed as the applied field increases further (Allahverdizadeh et al., 2012; Haiqing et al., 1993; Rajamohan et al., 2010c; Yalcintas and Coulter, 1995c). This might be attributed to the variations in the storage modulus and loss modulus of MR/ER fluids with magnetic/electric field. Li et al. (2005) employed rheometer to investigate pre-yield dynamic properties of a typical MR fluid under different levels of magnetic flux. They identified four field-induced regions, I, II, III, and IV, in the system. These regions were defined by three values of critical magnetic flux density, Bc_1, Bc_2, and Bc_3, as depicted in Figure 7. In the first region, the fluid, which was subjected to very low magnetic flux, exhibited Newtonian behavior and experienced coexisting of particles and random chains. The storage modulus was almost constant, while the loss factor increased slightly with magnetic flux. The storage modulus and loss factor increased significantly in the second region in which the MR fluid was a mixture of chains and random clusters. In the third region, the fluid experienced coexisting of the clusters and chains and the storage modulus and loss factor sharply increased and decreased, respectively. The last region contained saturated MR fluid, and rheological properties of the fluid showed no variation with the applied magnetic field. In this region, the particle chains formed stable clusters in the direction of applied magnetic field. These four regions may interpret variations in the storage modulus and loss factor of the MR/ER sandwich structures with respect to the applied field.

The adjustable stiffness and damping properties of MR/ER fluids enable adaptive sandwich structures to suppress unwanted vibration and decrease instability (Dwivedy and Srinivas, 2008; Tylikowski, 2002). Hu et al. (2006) suggested a significant decrease in the displacement amplitude and notable rightward shift in the resonant frequencies of MR-based sandwich beam structures, in response to the applied magnetic field. Both the stiffness and damping of the sandwich structures could be substantially varied by varying the applied magnetic field. The monotonic decrease in the displacement amplitude could be directly attributed to an increase in both the stiffness and damping of the structure with increasing magnetic field. It has also been reported that higher magnetic flux reduces sharpness of the peaks in frequency response function of the structure, which is attributed to an increase in the damping of the sandwich beam (Sun et al., 2003). Ying and Ni (2009) employed a cantilever sandwich beam with MRE core and supplemental mass under stochastic support motion excitation to model a smart composite wall with floor and equipment. The results show significant effect of the core layer in minimizing the velocity response of the sandwich beam.

Effects of different parameters on dynamic responses of MR/ER sandwich beams. Irrespective of applied magnetic/electric field, some other parameters such as material and thickness of the face layers (Allahverdizadeh et al., 2012, 2013a, 2014; Choi et al., 1992; Rezaeepazhand and Pahlavan, 2008a), fluid thickness (Mohanty, 2013; Rezaeepazhand and Pahlavan, 2008a; Yeh et al., 2004), boundary conditions (Allahverdizadeh et al., 2013a; Lee and Cheng, 1998; Yalcintas and Coulter, 1995a, 1995b, 1995c), external disturbances (Allahverdizadeh et al., 2012; Wei et al., 2007), modes of vibration (Lu and Li, 2007; Yalcintas and Coulter, 1995a), and temperature (Gandhi et al., 1989) may also change the natural frequencies, loss factors, and vibration amplitude of the MR/ER-based sandwich structures. Yeh and Shih (2005) suggested an increase in the loss factor of the ER sandwich structures with core layer thickness. The study of Rezaeepazhand and Pahlavan (2008a) showed that increasing the core layer thickness increased the settling time in transient response of the sandwich beam structure, which was attributed to increasing mass of the structure and decreasing the shear deformation in the core layer. This behavior might be attributed to low applied electric field employed in the study. It is worth noting that irrespective of boundary condition, the region of instability for the MR/ER sandwich beam under axial load starts at a higher frequency compared with the untreated structure (Dwivedy et al., 2009; Dwivedy and Srinivas, 2011; Nayak et al., 2012a). It should be noted that in these studies, the structure was subjected to an axial load consisting of constant static and harmonic variable loads. Furthermore, increasing MR/ER core layer thickness may increase critical dynamic loading of the sandwich structures (Dwivedy et al., 2009; Dwivedy and Srinivas, 2011; Nayak et al., 2012a; Tabassian and Rezaeepazhand, 2011, 2013; Yeh et al., 2004; Yeh and Shih, 2005, 2006a, 2006b). The instability region decreases with application of the static magnetic field over the structure (Nayak et al., 2012a; Yeh and Shih, 2006a, 2006b).

Allahverdizadeh et al. (2012) investigated the effect of face layer materials on dynamic characteristics of ER sandwich beam. They conducted a vibration analysis on the rotating ER sandwich beam with face layers of FGM, which was a mixture of ceramic and metal. Their study showed that increasing the FGM volume fraction index at constant rotating speed decreased (increased) the natural frequencies (loss factors) of the structure. It was attributed to lower stiffness of the metallic part compared to ceramic component. In fact, increasing the FGM volume fraction enhanced contribution of metallic part in face layer material and decreased stiffness of the structure. They also reported an increase and decrease in the natural frequencies and loss factors of the structure, respectively, as the rotating speed increased. This was also reported by Rajamohan (2013) and Rajamohan and Natarajan (2012) for rotating MR sandwich beam structures. Wei et al. (2006, 2007) related this phenomenon to increasing stiffness of the sandwich beam and instability of the suspended particles at higher rotor speed. It is worth noting that the vibration suppression capability of rotating ER fluid decreases by increasing the rotational speed, while it is not affected by rotating acceleration (Wei et al., 2007). The schematic and real image of a rotating ER fluid is shown in Figure 8. Nayak et al. (2014) investigated dynamic stability of rotating sandwich beam with MRE core layer. They reported significant improvement in stability of the system in response to increasing magnetic field, rotational speed, and ratio of hub radius to beam length. Yalcintas and Coulter (1995a) investigated the effect of geometry boundary condition on the natural frequencies and loss factors of ER sandwich beam and realized that the higher constrained boundary conditions elevated the natural frequencies and reduced the loss factors of the structure, which was due to small shear deformation experienced by the core layer. Similar results were also reported for MR sandwich beam by Rajamohan et al. (2010c).

Choi et al. (2010) illustrated frequency-dependent behavior in the loss factors of an MRE sandwich beam, experimentally and theoretically. They suggested relatively stronger influence of excitation frequency on the loss factors, under low level of magnetic flux density compared to that of higher magnetic flux. This might be attributed to relatively stronger adhesion of magnetic particles under the higher magnetic flux, where the vibration frequency effect became less significant (Li et al., 1999). Yalcintas and Dai (1999) reported a decrement in the loss factors of an MR-based sandwich structure with increasing the mode numbers. It was believed that the loss factor reduction was attributed to low shear deformation of the structure in higher frequencies. The results presented by Yalcintas and Coulter (1995b), however, show some exceptions in the clamped-free ER sandwich beam so that the loss factors corresponding to the first two modes were smaller than

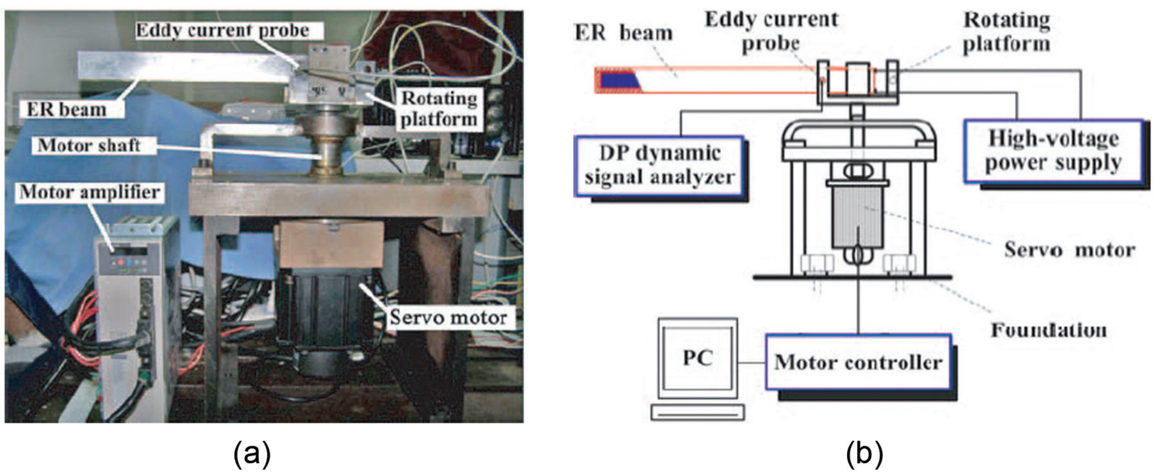


Figure 8. (a) Real image and (b) schematic diagram of a rotating ER fluid (Wei et al., 2007).

those of other modes. They attributed it to the mode shapes of the sandwich beam structure. The wavelength of clamped-free sandwich beam corresponding to the first mode was smaller than those of the other end conditions, which resulted in reducing the shear deformation of the core layer. It is widely reported that the loss factors corresponding to the lower modes of MR/ER-based sandwich structures increase initially with the excitation frequency and then decrease as the excitation frequency increases further (Yalcintas and Coulter, 1995b, 1995c). The dynamic responses of ER sandwich structures are also highly temperature dependent. Gandhi et al. (1989) outlined less significant increment of the resonant frequencies and damping ratios of ER sandwich beam with respect to the applied electric field at higher temperature, which was attributed to adverse effect of the temperature on electro-viscous phenomenon of the ER fluids.

Non-linear analysis of MR/ER sandwich beam structures. While most of the studies on dynamic characteristics of MR/ER sandwich structures assumed the core layer to operate in the pre-yield region, the non-linear behaviors of the MR/ER sandwich structures have also been reported (Allahverdizadeh et al., 2014; Lee and Cheng, 1998; Phani and Venkatraman, 2005; Qiu et al., 1999; Rezaeepazhand and Pahlavan, 2008a). Lee and Cheng (1998) showed that the effect of applied electric field on the non-linear natural frequencies and loss factors of the ER sandwich structure decreased with increasing the vibration amplitude. Phani and Venkatraman (2005) suggested a decrease in the loss factors of ER sandwich beam with vibration amplitude, in the non-linear region. They believed that the inverse relation between the loss factor and vibration amplitude indicated coulomb friction type of damping in the core layer. Rezaeepazhand and Pahlavan (2008a) employed Bingham model to investigate transient response of a sandwich beam with ER core layer in the post-yield regime. Their results showed negligible contribution of the viscous damping component of the core layer in the damping properties of the structure, compared to the damping component associated with applying electric field (coulomb damping). Allahverdizadeh et al. (2014) compared dynamic characteristics of an ER sandwich beam assuming ER fluid in the pre- and post-yield regions. They reported an increase in the non-linear frequency ratio ($\omega_{non-linear}/\omega_{linear}$) of the ER sandwich beam with increasing vibration amplitude, while the structure was under electric field of 2 kV mm^{-1}. They attributed this trend to typical hardening behavior of the structure in large vibration amplitude. Furthermore, they suggested a decrease in the loss factor ratio of the structure by increasing the vibration amplitude, which was due to the smaller force required to break chains of dielectric particles in the ER fluid. Their study showed that the non-linear frequency or loss factor ratios were not equal to 1, even at very low range of vibration amplitude. This verifies non-linear behavior of MR/ER fluids under high magnetic/electric field, irrespective of shear strain amplitude.

Haiqing et al. (1993) employed ER fluid as a complex spring under a cantilever beam, as depicted in Figure 9. The results showed that increasing the applied electric field increased the stiffness and damping properties of the structure in non-linear fashion so that the frequency response function curve changed its shape in different amplitudes of excitation force. This study illustrated that the vibration characteristics of ER-based sandwich structure could be considered to be linear in very low ($E = 0$ kV mm^{-1}) and high ($E = 5$ kV mm^{-1}) electric field strengths, while behavior of the system was non-linear at medium range of electric field ($E = 3$ kV mm^{-1}). They related the non-linearity to simultaneous contributions of the pre-yield and post-yield deformations on the system vibration. Phani and Venkatraman (2005) outlined that the linear behavior of the structure in low electric field strength indicates dominate contribution of the elastic face layers in the flexural dynamics relative to the ER fluid core layer.

Disagreements between the theoretical and experimental results on MR/ER sandwich beam structures. In the study of MR/ER sandwich beam structures, some discrepancies between theoretical and experimental results may be observed. Don and Coulter (1995) reported significant disagreement between the theoretical and experimental results on dynamic characteristics of an ER sandwich beam. Figure 10 shows the variations in lower two natural frequencies of the sandwich ER beam with electric field, obtained from the experiment and RKU model. They attributed discrepancy of the results to overfilling of the core layer. Their study showed that overfilling increased mass of the structure and reduced the effective electric field, which was due to an increase in the sandwiched layer spacing. Furthermore, they found that extensive testing caused the liquid component of the ER material to seep through corners of the structure and increase the volume fraction of particles suspended in the fluid which resulted in changing rheological behavior of the fluid in response to the electric field. Lee and Cheng (1998) considered complexities in accurate modeling of the boundary condition as a source of error in their study. Several studies have considered inaccurate characterization of MR/ER fluids as a source of error in the experimental studies (Coulter and Duclos, 1990; Coulter et al., 1989, 1993c; Don, 1993; Yalcintas and Coulter, 1995b). Yalcintas and Dai (2004) related the disagreement between theory and experiment to neglecting the effect of sealant in the mathematical modeling, non-uniformity of magnetic/electric field, and additional unwanted constraints existing in the experimental setup. The sealant increases the stiffness and damping of the structure. Choi et al. (2010) attributed the discrepancies to coupling between the sandwich beam and the test rig, bonding between skin and core layers, and possible non-linearity in the sandwiched layer.

The study conducted by Sun et al. (2003) on MR sandwich beam involved significant disagreement between the theoretical and experimental results. This deviation was more noticeable, when the structure vibrated in the low vibrational modes. They believed that deviation of the results might be attributed to different layouts of the sandwich beam in the experiment and theory. In the theoretical analysis, it was assumed that the structure was placed between two permanent magnets horizontally. In the experiment, in order to eliminate the effect of bending of the beam due to its weight, the structure was placed perpendicularly between permanent magnets. Figure 11 shows the horizontal and vertical configurations of MR sandwich beams. Yalcintas and Dai (2004) and Lara-Prieto et al. (2010) have also addressed the concerns regarding horizontal position of the MR sandwich structure. Wei et al. (2007) employed ER fluid to attenuate vibrations of the robot arm for IC packaging. They modeled the

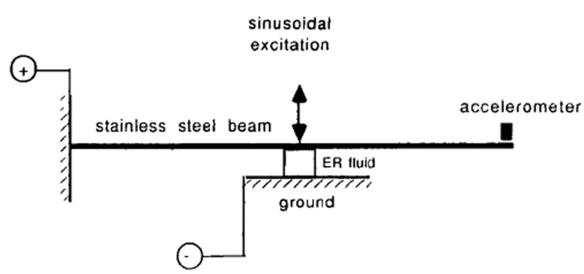


Figure 9. ER fluid as complex spring applied to a cantilever beam (Haiqing et al., 1993).

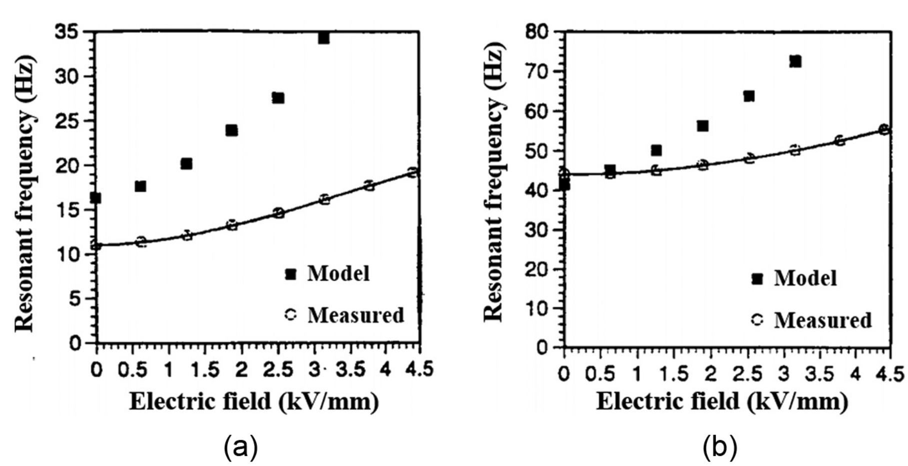


Figure 10. Variations in the (a) first and (b) second natural frequencies of ER-based sandwich beam with applied electric field (Don and Coulter, 1995).

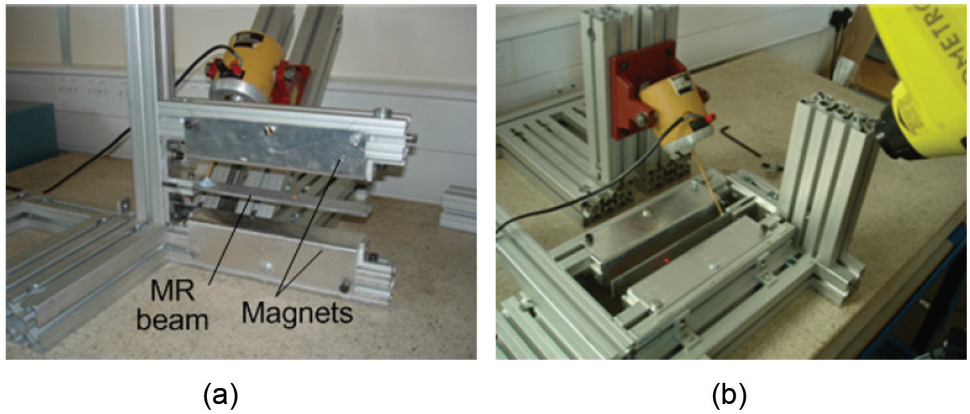


Figure 11. (a) Horizontal and (b) vertical positions of MR sandwich beam (Lara-Prieto et al., 2010).

arm as a rotating sandwich beam containing ER fluid. Their study revealed that although there was a qualitative agreement between the theoretical and experimental results, the predicted resonant frequencies and loss factors were higher and lower than experimental results, respectively. They attributed these differences to fabrication of the test beam and inaccuracies in the boundary conditions. Although theory assumes uniform thickness of three layers, it was not achievable in the experiment due to bending of the elastic layers and non-uniform adjustments of the sealant material.

Dynamic characteristics of fully treated MR/ER sandwich plates and shells

MR/ER sandwich plates with rectangular face layers

Compared to multi-layer MR/ER beam structures, fewer studies have been reported on sandwich plates containing MR/ER fluids as the core layer. The studies on vibration behavior of sandwich plates have been mostly limited to ER-treated ones (Coulter et al., 1993a, 1993b; Rezaeepazhand and Pahlavan, 2008b; Yeh and Chen, 2004, 2005) and very few studies have investigated dynamic properties of MR sandwich plates (Pranoto et al., 2004; Ramamoorthy et al., 2014; Yeh, 2013; Ying et al., 2012, 2014). So far, no experimental study has been reported on the vibration analysis of fully treated MR-based sandwich plates, which is perhaps due to various challenges associated with providing a uniform magnetic flux over the structure. Review of literature shows that appropriate applications of MR/ER core layers increase controllability (Cho et al., 2005) and stability (Rahiminasab and Rezaeepazhand, 2013) and decrease vibration amplitude (Hasheminejad and Maleki, 2009) of the sandwich plates, remarkably. Rahiminasab and Rezaeepazhand (2013) employed ER fluid to change flutter boundaries of sandwich structures. They reported significant effect of ER core layer on aerodynamic stability of the sandwich plates. In fact, application of electric field caused flutter to occur at higher aerodynamic pressure, which was mostly attributed to increasing stiffness of the structure. A similar study was conducted by Hasheminejad and Motaaleghi (2014) to investigate active flutter suppression of sandwich shell containing ER fluid, under axial supersonic gas flow.

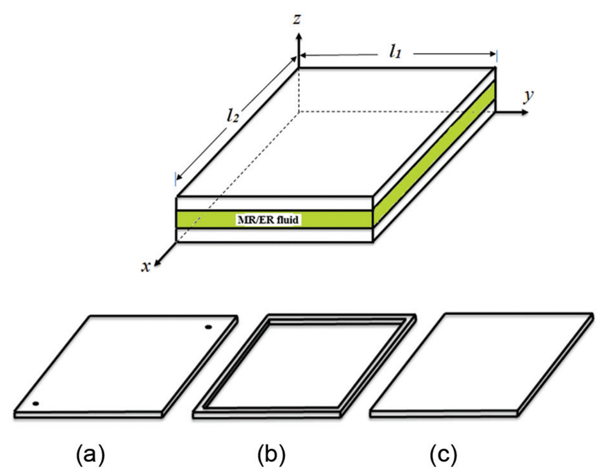


Figure 12. A sandwich plate with MR/ER fluid core layer consisting of (a) constraining top layer, (b) sealant spacer, and (c) base layer (Eshaghi et al., 2015a).

Mathematical modeling of MR/ER sandwich plates. MR/ER sandwich plate, as depicted in Figure 12, comprises base layer, sealant spacer, constraining layer, and MR/ER fluid as the core layer. The assumptions employed in the analysis of adaptive sandwich beams (no slippage between the layers, uniform transverse displacement through the thickness, and negligible normal stress and transverse shear strain in the core and face layers, respectively) are also applicable for the analysis of the MR/ER plate structures (Choi et al., 1999; Hasheminejad and Maleki, 2009; Lu and Meng, 2006;

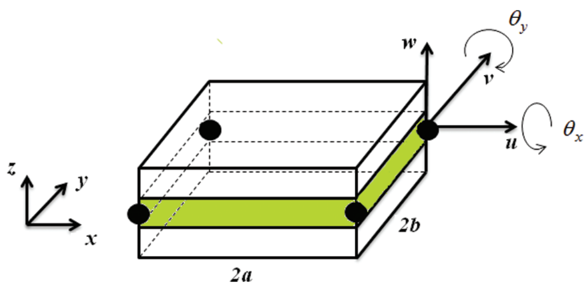


Figure 13. Two-dimensional sandwich plate element (Eshaghi et al., 2015a).

Narayana and Ganesan, 2007). Due to small thickness ratio of the face layers, classical plate theory was widely employed to identify displacement field in the face layers (Hasheminejad and Maleki, 2009; Yeh and Chen, 2004, 2005, 2007). The displacement profile of the core layer was also obtained using compatibility conditions of three layers. Application of FE method has been extensively reported to derive governing equations of motion of MR/ER sandwich plates under different geometry boundary conditions (Cho et al., 2005; Lu and Meng, 2006; Yeh, 2007b, 2010b, 2011a; Yeh et al., 2009).

Eshaghi et al. (2015a, 2015c) employed a sandwich plate element, as depicted in Figure 13, to derive governing equations of motion of an MR sandwich plate. The element consisted of four nodes with 10 DOF (longitudinal displacements, transverse displacement, and slopes about x- and y-axis for the top and bottom layers), per node. Since the displacement field of the core layer was obtained from those of the top and bottom layers, the DOFs of each node were associated with those at the top and bottom layers. The transverse and longitudinal displacements of the element were expressed in terms of the nodal displacement vector, $q(t) = \{u_{0i}^{(1)}, v_{0i}^{(1)}, u_{0i}^{(3)}, v_{0i}^{(3)}, w_i^{(1)}, w_i^{(3)}, \theta_{xi}^{(1)}, \theta_{xi}^{(3)}, \theta_{yi}^{(1)}, \theta_{yi}^{(3)}\}^T$ for $i = 1, 2, 3, 4$, and the shape function vectors. Hasheminejad and Maleki (2009) employed Lagrangian equation and developed an exact closed form solution to identify dynamic responses of an ER sandwich plate. The solution was only applicable for simply supported structures.

Developing theoretical model representing dynamic characteristics of adaptive sandwich plates requires accurate characterization of MR/ER fluids contained in the core layer. Cho et al. (2005) and (1999) employed rheometer to characterize ER fluids employed in the sandwich ER plate structures. Choi (2000) adopted the model developed by Choi et al. (1990), which was based on treating cantilever beam as an SDOF system, to analyze ER sandwich plate. The ER models developed by Don (1993) and Yalcintas and Coulter (1995c) have been widely used to identify the complex shear modulus of ER fluids employed in the core layer of the sandwich plates (Hasheminejad and Maleki, 2009; Yeh and Chen, 2004, 2005, 2006, 2007). Yeh (2013) utilized the quadratic functions proposed by Rajamohan et al. (2010c) to investigate dynamic characteristics of MR sandwich plate. Aguib et al. (2014) conducted an experiment using visco-analyzer to identify rheological properties of an elastomer employed in an MRE sandwich plate.

Dynamic responses of MR/ER sandwich plates. The studies on MR/ER sandwich plates suggest an increase in the resonant frequencies of the structures with increasing the applied magnetic/electric field (Choi et al., 1999, 2001, 2005; Hasheminejad and Maleki, 2009; Yeh, 2007a, 2007b, 2013). In particular, the effect of applied field strength on the lower modes resonant frequencies was more pronounced compared to those of the higher modes (Hasheminejad and Maleki, 2009). While the natural frequencies of the MR/ER-based sandwich plates increase with applied magnetic/electric field, the variations in the loss factors with respect to the applied field do not follow the same trend. Figure 14(a) shows that similar to sandwich beams, the loss factor of ER sandwich plates increases at low electric field strength, reaches a peak at intermediate field magnitude, and subsequently decreases as the field strength is further enhanced (Hasheminejad and Maleki, 2009). This behavior has also been reported for MR sandwich plate (Yeh, 2013).

The loss factor peak may shift to the higher or lower magnetic/electric field amplitude in the structures with different face or core layer thicknesses, aspect ratios, MR/ER fluids, and excitation frequencies. For instance, Hasheminejad and Maleki (2009) illustrated that higher aspect ratio of an ER-based sandwich plate shifted the loss factor peak to the higher electric field magnitudes. Yeh and Chen (2007) observed a decrease in the loss factor of ER plate as the electric field increased, as depicted in Figure 14(b). In this case, the loss factor peak shifted to low electric field strength. It is worth noting that although it is expected that application of magnetic/electric field over the MR/ER sandwich plates always enhances the stiffness and damping properties of the structures and suppresses undesired vibration, Hasheminejad and Maleki (2009) concluded that applying an electric field does not necessarily lead to improving vibration response of an ER sandwich plate. In fact, corresponding to a specific frequency of excitation, there was an optimal electric field which caused minimum displacement amplitude. In other words, vibration suppression capability of MR/ER sandwich plates is frequency dependent (Lu and Meng, 2006). On the other hand, an anti-optimal electric field could result in maximum displacement amplitude in the structure. Accordingly, inappropriate application of magnetic/electric field may degrade vibration control

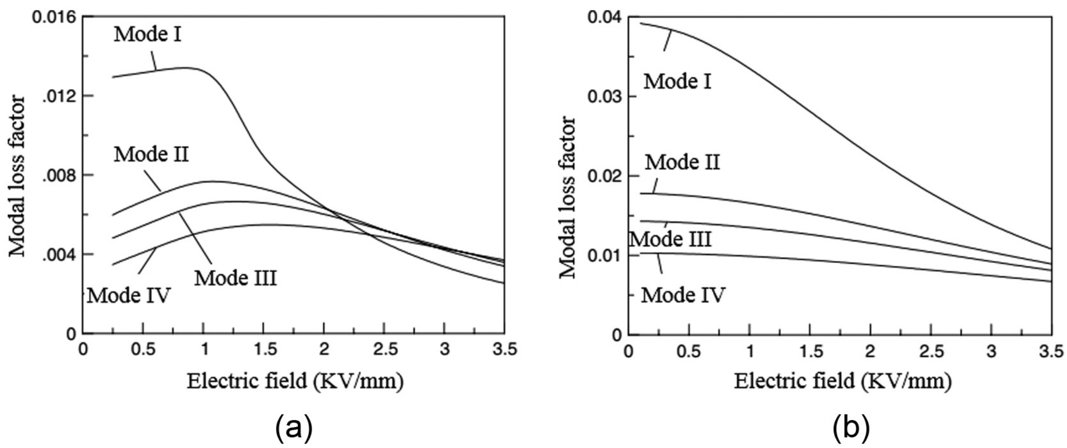


Figure 14. (a, b) Variations in modal loss factors corresponding to lower four modes of a sandwich plate with two different ER fluids (Yeh and Chen, 2007).

performance of the MR/ER sandwich plates, significantly (Yalcintas and Coulter, 1995b).

While magnetic/electric field strength highly affects the resonant frequencies and loss factors of sandwich plates, some other parameters such as face layer geometry (Hasheminejad and Maleki, 2009; Narayana and Ganesan, 2007), core layer thickness (Yeh, 2007a, 2007b, 2013), boundary conditions (Narayana and Ganesan, 2007), and excitation frequency (Lu and Meng, 2006) influence modal parameters of the structures, remarkably. The significance of these factors on dynamic responses of the sandwich plates is relatively different. Hasheminejad and Maleki (2009) reported more noticeable effect of electric field strength on vibration suppression of ER sandwich structure in comparison with increasing the ER core layer thickness. They also suggested direct and inverse effects of aspect ratio on the natural frequencies and loss factors of ER sandwich plate, respectively. In other words, the natural frequencies (modal loss factors) increase (decrease) with increasing the aspect ratio. Narayana and Ganesan (2007) investigated the effect of boundary condition on dynamic responses of sandwich plate containing ER fluid. They concluded that increasing constraints on the four edges of sandwich plate increased natural frequencies and decreased modal loss factors. Cantilever and all clamped edge plate showed the maximum and minimum loss factors, respectively. They also found that the sandwich plate with viscoelastic as the core layer was stiffer and showed superior damping properties compared to ER-based sandwich plate, under all boundary conditions except Clamped-Free-Free-Free (CFFF) and Clamped-Clamped-Free-Free (CCFF). In these cases, the conventional viscoelastic core exhibits poor damping for the first few modes, when compared to the ER plate. It should be noted that C and F represent clamped and free ends, respectively.

Core layer thickness plays a significant role in dynamic characteristics of the sandwich plate. The core thickness might significantly enhance vibration controllability of the structure, under high levels of applied field (Vaičaitis et al., 2007). Increasing the core layer thickness may decrease the natural frequencies (Yeh and Chen, 2004) and increase the modal loss factors (Narayana and Ganesan, 2007). A decrease in the resonant frequencies signifies that the effect of thickening the core layer on mass of the structure is more significant than its stiffness (Mohammadi and Sedaghati, 2012c). Yeh and Chen (2004) suggested that application of strong electric field may cause the natural frequencies of the ER sandwich plate to increase continuously as the thickness ratio of the ER layer increases. Furthermore, Yeh and Chen (2007) showed that under high electric field strength, the modal loss factor of ER sandwich plate increased with increasing ER layer thickness. However, under low electric field strength, it decreased initially and then increased, by a further increase in ER layer thickness. Yeh and Chen (2005) reported a decrease in the dynamic stability regions of the sandwich plate with increasing ER layer thickness. Rahiminasab and Rezaeepazhand (2013) suggested a decrease in the critical aerodynamic pressure of sandwich ER plate with an increase in ER core thickness, while increasing the constraining layer thickness caused a reverse effect.

Sandwich structures with annular plate, skew plate, and shell face layers

Although rectangular sheets have been widely employed to serve as the constraining and base layers for MR/ER sandwich plates, application of annular (Yeh, 2007a, 2007b, 2010a, 2010b, 2011a, 2012; Yeh et al., 2009) and skew plates (Narayana and Ganesan, 2007) has also been reported. The effects of different parameters such as magnetic/electric field, core layer thickness, aspect ratio, and boundary condition on the annular or skew plates are similar to those of the

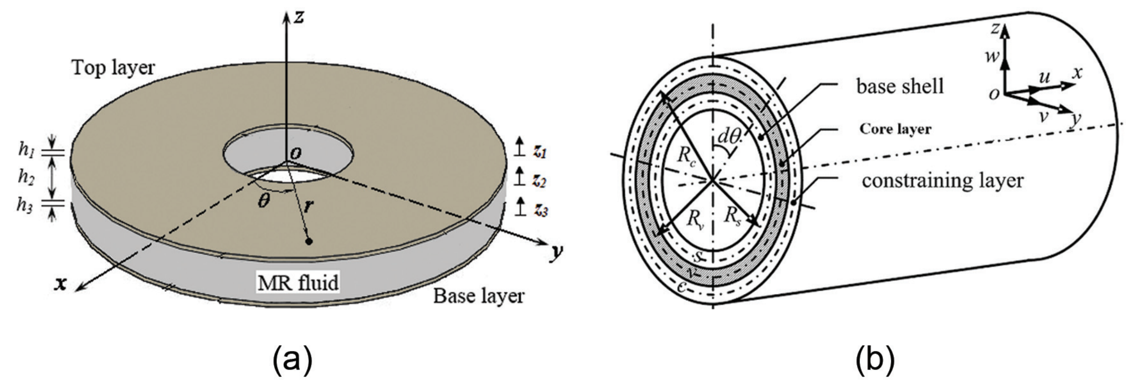


Figure 15. Sketch of sandwich (a) annular circular plate and (b) shell structure (Wang and Zheng, 2013).

rectangular sandwich plate structures. Furthermore, few studies have addressed dynamic characteristics of sandwich shells with MR/ER fluids as the core layer (Hasheminejad and Motaaleghi, 2014; Mikhasev et al., 2011; Mohammadi and Sedaghati, 2012b, 2012c; Tylikowski, 2000; Yeh, 2011b). Figure 15 shows two typical annular circular and shell sandwich plates. Mohammadi and Sedaghati (2012b) presented non-linear vibration analysis of a sandwich shell with ER fluid as the core layer. They proposed a new notation referred to as H-notation in the FE analysis to reduce computational costs. The results demonstrated hardening type in the non-linear behavior of the sandwich structure so that increasing the amplitude of excitation increased the resonant frequency ratio ($\omega_{non-linear}/\omega_{linear}$). The rate of variations, however, depended on boundary condition and core layer thickness, significantly. Yeh (2011b) adopted FE method to identify dynamic characteristics of multi-layer shell structure with ER fluid. The dynamic responses of the shell structure, however, follow the same trend as sandwich plate structures (Yeh, 2013), under different electric field levels, core layer thicknesses, and modes of vibration. So far, no experimental study on MR-based sandwich shell has been reported, which might be related to challenges in providing magnetic flux over the structure. Mikhasev et al. (2011) presented the only theoretical study on dynamic characteristics of non-circular cylindrical shell with MR fluid as the core layer. The structure consisted of N transversely isotropic layers and MR core layers were sandwiched between the elastic layers. Although the results demonstrated the correlation between vibration suppression capability of the structure and material properties of layers, the effect of MR layer thickness in vibration control of the structure was highlighted. Mikhasev et al. (2014) investigated the effect of magnetic field on eigenmodes of MRE-based sandwich shell structure. They considered physical properties of MRE layer to be function of magnetic field induction and curvilinear coordinate. Their study suggests localization of eigenmodes corresponding to low-frequency spectrum of the structure in response to the applied magnetic field.

Applications of MR/ER sandwich plates

While the main application of adaptive MR/ER-based sandwich plates was to provide adjustable stiffness and damping properties to suppress unwanted vibration of the base layer, review of literature also suggests some other applications (Choi et al., 2001; Pranoto et al., 2004). Choi et al. (2001) investigated the effect of ER sandwich plate on noise control. They proposed an acoustic cavity comprising five acrylic sheets and ER plate as the sixth face of the cavity. A loud speaker generated sound pressure from outside of the cabin. The speaker was excited with sweep sine signals from the function generator through the power amplifier. A microphone was used to measure the sound level inside the cavity through a small hole in the bottom of the cavity. The fuzzy control algorithm was adopted to attenuate sound transmission from the speaker into the cabin. The experimental setup is shown in Figure 16. The results signified remarkable effect of ER plate on sound pressure attenuation in the cabin. Application of controlled electric field reduced the pressure level by 20 and 19 dB at 62 and 98 Hz, respectively. Hasheminejad and Shabanimotlagh (2010) investigated the effect of MRE on sound insulation improvement of a sandwich plate containing MRE as the core layer. Although application of magnetic field caused no significant improvement in sound insulation at low frequencies, the effect on intermediate- and high-frequency regions was noticeable.

Harland et al. (2001) inserted ER sandwich plate into a part of vibrating structure to attenuate vibration transmission. In their experiment, a sandwich ER plate was glued between two sections of a clamped–clamped Perspex beam to suppress vibration transmission, as depicted in Figure 17. The advantage of proposed insert over traditional passive dampers was to provide

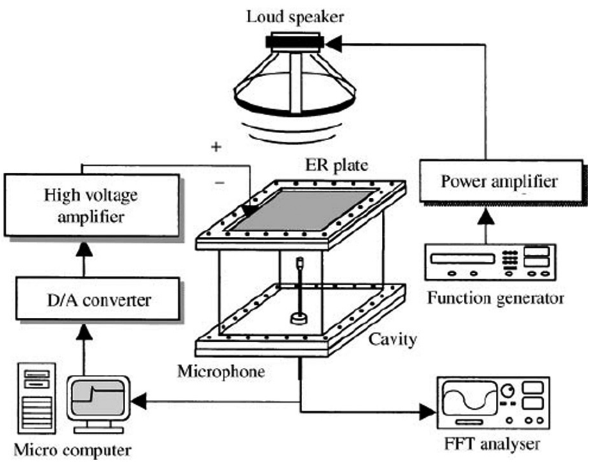


Figure 16. Experimental setup for noise control (Choi et al., 2001).

variable properties and have negligible effect on integrity and mass of the structure. The Perspex beam was excited on one side of the insert while the vibration was measured on both sides of the sandwich insert. The results suggested significant effect of the insert in reducing vibration transmission in the main structure.

Figure 18 shows a shear mode MR damper designed by Pranoto et al. (2004), which is applicable for vibration suppression of large flexible structures such as aircraft wings. The damper comprised a thin rectangular box, number of thin plates with slits, and MR fluid which was contained in the box. Permanent magnets attached to the box surface provided magnetic flux over the structure and solidified the MR fluid to resist against shear deformation due to plates sliding. The advantage of the proposed damper over ordinary hydraulic damper was to work for small displacement. In contrast to rubber damper, which shows poor performance in low-frequency region, the proposed damper could generate large damping force in the low frequencies. The resisting force due to shear motion was almost constant and frequency-independent, which was the main difference of the proposed damper with piston-type ones. The proposed thin and light damper worked passively and needed no energy for activation. Pranoto et al. (2004) applied the damper on a vibrating wing and reported 90% reduction in amplitude of vibration corresponding to the first (bending) and second (torsional) modes of the wing.

Dynamic characteristics and optimum design of partially treated MR/ER sandwich structures

Partially treated sandwich beam, plate, and shell structures

A survey of literature shows substantial increase in the stiffness and damping properties of the sandwich structures, fully treated with the MR/ER fluid layer, with increasing applied field. The fully treated sandwich structures, however, result in higher mass due to high weight density of the fluid and pose some practical challenges in implementing the MR/ER fluid layer (Kciuk and Turczyn, 2006). Furthermore, application of a uniform magnetic/electric field over the entire structure poses difficult challenges. Partial MR/ER fluid treatments would thus be desirable, particularly when applied to optimal locations to achieve maximum controllability with relatively small size treatment and low energy consumption. Partially treated sandwich structures comprise elastic face layers and partial MR/ER segments as the core layer. Figure 19 represents a partially treated MR sandwich plate in which MR fluid is located at the corner of core layer. Those parts of the core layer not covered by the MR/ER treatments (untreated regions) may be filled by other materials. Haiqing and King (1997) and Lu and Li (2007)

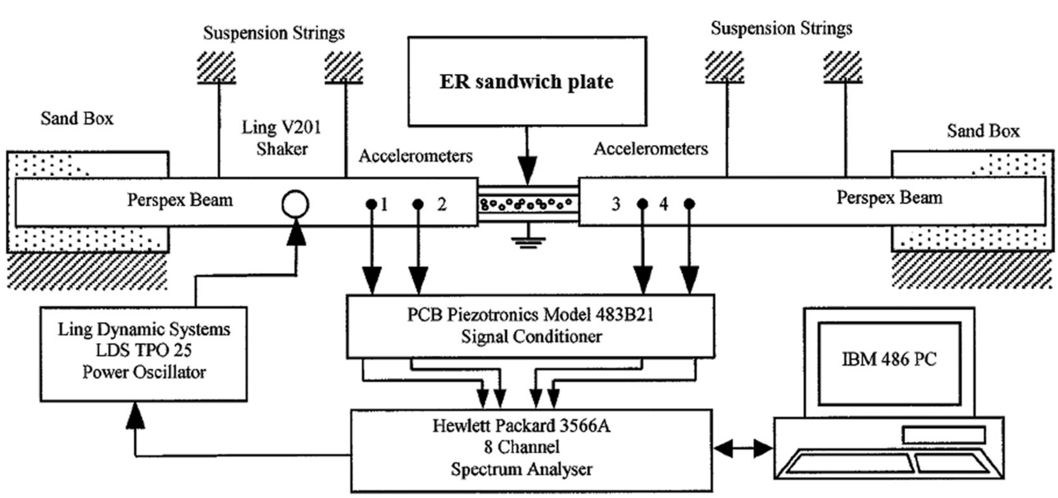


Figure 17. Experimental setup (Harland et al., 2001).

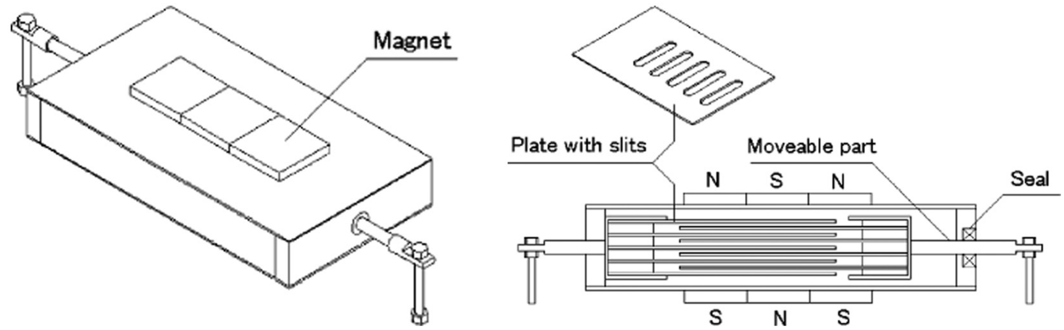


Figure 18. Geometry of MR damper (Pranoto et al. 2004).

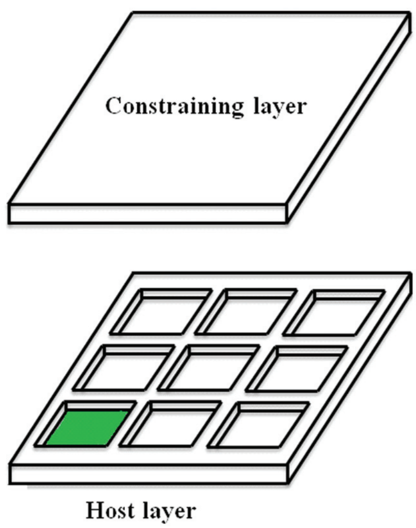


Figure 19. Partially treated MR sandwich plate (Eshaghi et al., 2015b).

considered untreated parts in the core layer of partially treated ER sandwich beam to be filled by air. Oyadiji (1996) and Choi et al. (1999) employed the same layout of the core layer to fabricate partially treated sandwich plate. Rajamohan et al. (2010a) proposed a sandwich beam of aluminum face layers and partially treated MR fluid as the core layer. The remaining segments of the core layer were considered to be of aluminum material. In order to suppress vibration amplitude of a cross-ply elastic composite laminate, Panah and Hasheminejad (2010) proposed a partial ER fluid segment attached to the base layer. The top layer was assumed to be as the same size of the ER segment. Mohammadi and Sedaghati (2012c) investigated dynamic responses of a partially treated sandwich shell structure. Unconstrained viscoelastic material was employed at boundaries and untreated locations to seal ER fluid. The thickness of the viscoelastic layer was considered to be equal to the thickness of the ER core and constraining layers. The logic behind choosing unconstrained viscoelastic material was to achieve more dependency of the structural loss factor on the constrained ER fluid.

Dynamic responses of partially treated sandwich structures. The properties of partially treated MR/ER sandwich structures are strongly affected by different fluid and structural related parameters such as core layer thickness, complex shear modulus of the fluid, applied field strength, face layer geometry, and boundary conditions. Furthermore, the number and location of the fluid treatments contribute significantly in the dynamic responses of the sandwich structures. In the absence of applied magnetic/electric field, partial treatments yield relatively lower natural frequencies of the sandwich structure compared to those of the untreated ones, irrespective of the configuration, end conditions, and modes of vibration (Rajamohan et al., 2010a). That is simply due to higher mass of the fully treated structure. Increasing the applied field generally results in an increase in the natural frequencies (Choi et al., 1999; Oyadiji, 1996). Haiqing and King (1997) reported reduction in the resonant frequencies of a partially treated sandwich ER beam in response to the electric field. The partial treatment was located at the middle of the clamped–clamped beam. They attributed decrease in the natural frequencies of the structure to solidification of ER fluid subjected to the electric field, which resulted in transmission of vibration energy applied to the upper steel face layer to the lower one. Oyadiji (1996) investigated the effect of ER segments' locations on the natural frequencies of a cantilever partially treated ER plate. Silicon rubber was employed to partition the core layer to five cavities parallel to the clamped edge. A constant electric field strength of 2 kV mm^{-1} was provided over the structure. Treating the first cavity (closed to the clamped edge) increased the natural frequencies, compared to those of the untreated structure. The increase was attributed to significant effect of electric field on the stiffness of the ER fluid compared to the mass effect. Treating the second cavity decreased the resonant frequencies which signified significance of the mass effect. This study suggested that treating 60% of the core layer by ER fluid resulted in minimum vibration amplitude of the structure. Rajamohan et al. (2010a) investigated the effect of the

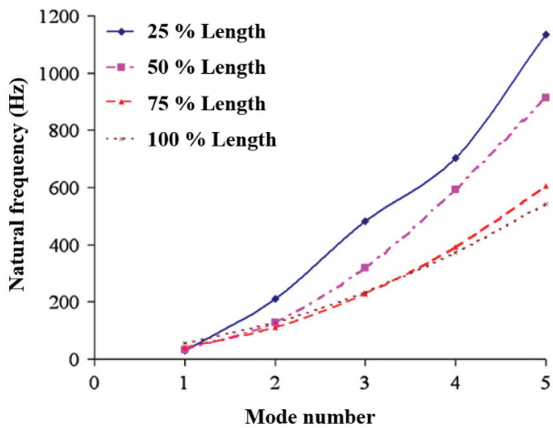


Figure 20. Variations in lower five natural frequencies of simply supported MR sandwich beam with MR fluid length (Rajamohan et al., 2010a).

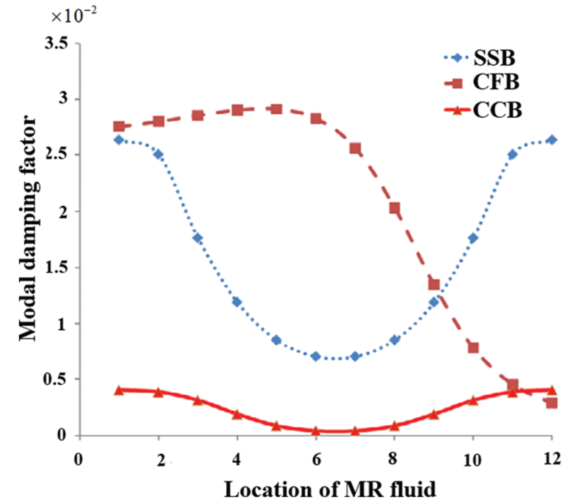


Figure 21. Variations in modal loss factor of MR sandwich beams with MR location under different boundary conditions (Rajamohan et al., 2011).

variations in the portion of core layer occupied by MR fluid on the natural frequencies of a simply supported partially treated MR beam, under magnetic flux density of 50 mT. The simulations were performed by considering MR fluid treatments over 25%, 50%, 75%, and 100% of the beam length. Figure 20 shows the variations in the resonant frequencies of the structure with different lengths of the treatment corresponding to the first five modes. Increasing volume of the MR fluid increased the mass and stiffness of the structure. In the lower modes, the effect of MR volume on the mass and stiffness was almost the same; hence, no significant variations in the resonant frequencies were observed. On the other hand, in the higher modes, the effect of MR volume on the mass was more pronounced compared to that of stiffness; thus, the natural frequencies decreased by increasing MR length. Panah and Hasheminejad (2010) suggested that the natural frequencies of partially treated sandwich plate subjected to low electric field strength (1 kV mm^{-1}) were lower than the untreated structure. Applying high electric field (3 kV mm^{-1}), however, enhanced the resonant frequencies compared to the untreated plate.

It is widely reported that the loss factors of fully treated sandwich structures are generally higher than those of the partially treated ones (Rajamohan et al., 2010a; Joshi, 2012). That is attributed to lower dissipated energy of the latter structure. However, some studies reported higher loss factor of partially treated sandwich structures compared to fully treated ones, under specific boundary conditions and geometries (Haiqing and King, 1997; Mohammadi and Sedaghati, 2012c; Panah and Hasheminejad, 2010; Rajamohan et al., 2010a). Haiqing and King (1997) suggested less significant effect of partial treatments on the loss factor of partially treated ER beam compared to natural frequencies. Rajamohan et al. (2011) investigated the effect of MR locations on the modal damping factors for five modes of partially treated sandwich beam, under SSB, clamp (CCB), and cantilever boundary conditions. They divided core layer into 12 portions and treated different portions. Figure 21 shows the variations in the fundamental modal damping factor of the beams with MR location. The results suggest that optimal location of MR fluid resulting in maximum loss factor of the structure strongly depends on the boundary condition.

Panah and Hasheminejad (2010) illustrated the effects of electric field strength and ER patch size on the loss factors of a partially treated ER plate. They showed that employing a large ER segment under high electric field or small segment under low electric field strength increased the loss factors of the structure. The results also showed noticeable effect of shear deformation in vibration suppression capability of the ER sandwich structures. In other words, the MR/ER fluid segments may adjust the stiffness and damping properties of the partially treated structure in a wide range if they experience noticeable shear strain. Tylikowski (2000) analyzed vibration responses of a cylindrical shell partially treated with ER fluid, under different boundary conditions. The study concluded more significant variations in the modal parameters of the free–free structure, compared to those of sandwich shell with fixed-free boundary condition, in response to the electric field, which was due to higher shear deformation in the former structure.

Partial activation of the core layer of MR/ER sandwich structures

The effects of partially activation of MR/ER fluids on dynamic responses of the sandwich structures have been investigated in some studies (Cho et al., 2005; Choi et al., 1999; Lara-Prieto et al., 2010; Yalcintas and Coulter, 1998). Choi et al. (1999) fabricated a four-partitioned ER plate to investigate the effect of intensity and area of applied field on the natural frequencies and mode shapes of the structure. The sandwich plate was fully treated by ER fluid and each quarter of the core layer could be activated individually. The results suggested that energizing larger area of the core layer resulted in noticeable suppression of the mode shapes. Cho et al. (2005) proposed an ER-based sandwich plate with multi-electrode configuration to provide partial activation of the ER fluid. In other words, the structure was fully treated by ER fluid while the electric field was applied to partial area of the core layer. The study suggested an increase in the natural frequencies and loss factors of the structure as the area of active electrodes increased. Furthermore, activation of regions experiencing significant shear deformation yielded noticeable variations in the modal parameters of the structure. The study concluded that optimal activation of the fluid provided superior ratio of vibration suppression to consumed energy compared to a fully activated ER plate. Lara-Prieto et al. (2010) conducted an experiment to investigate the effect of partial activation of core layer on dynamic characteristics of a cantilever sandwich PET beam structure containing MR fluid as the core layer. It was observed that the natural frequency of the beam decreased as the activated area moved away from the clamped end of the cantilever structure. These results were in agreement with the theoretical study conducted by Yalcintas and Coulter (1998) in which they reported a decrease in the resonant frequencies of a simply supported ER beam when only the central region of the beam was activated.

Optimum design of partially treated sandwich structures

The advantage of partially treated over fully treated sandwich structure is to realize an appropriate layout with minimum treatments to obtain almost the same performance as the fully treated one. The design of a partially treated MR/ER sandwich structure requires finding an appropriate configuration of the treatments so that it yields the maximum controllability and vibration suppression capability of the structure in terms of stiffness and damping. For this purpose, different optimization problems on MR/ER sandwich structures have been formulated (Mohammadi and Sedaghati, 2012c; Rajamohan et al., 2010b; Snamina, 2011). Rajamohan et al. (2010b) proposed three optimization problems to investigate the effect of MR layout on dynamic responses of partially treated MR beam. In the first case, the objective function was formulated to find the optimal locations of MR treatments resulting in maximum modal damping factors corresponding to the first five individual modes of vibration, such that:

Case 1.

$$\text{Maximize} f_1(X) = \eta_d \frac{\sum_{e=1}^{n} \phi_e^{(r)} k_e \phi_e^{(r)}}{\phi^{(r)T} K \phi^{(r)}}, \qquad (13)$$
$$r = 1, \ldots, 5 \quad \text{subjected to } 0 < X \leq N$$

where $\phi^{(r)}$ is the rth mode shape vector; $\phi_e^{(r)}$ is the vector extracted from $\phi^{(r)}$ representing the displacement of eth MR fluid element in the core layer; K and k_e denote system and element stiffness matrices, respectively; η_d is the structural loss factor of MR fluid; and n and N are the number of MR fluid segments and FEs of sandwich beam, respectively. The design variable, X, is the location of the MR fluid segments in the core layer of the sandwich beam. The objective function corresponding to Case 2 was considered to maximize summation of the modal damping factors associated with the first five modes.

Case 2.

$$\text{Maximize} f_2(X) = \sum_{r=1}^{5} \left\{ \eta_d \frac{\sum_{e=1}^{n} \phi_e^{(r)} k_e \phi_e^{(r)}}{\phi^{(r)T} K \phi^{(r)}} \right\} \qquad (14)$$

Since the modal loss factor corresponding to the fundamental mode of sandwich beam overweighed those of the other modes, it was observed that the results obtained in Case 2 were identical to those obtained in Case 1. Consequently, the third objective function was considered to be logarithmic damping factors corresponding to the first five modes, such that:

Case 3.

$$\text{Maximize} f_3(X) = \sum_{r=1}^{5} \ln \left\{ \eta_d \frac{\sum_{e=1}^{n} \phi_e^{(r)} k_e \phi_e^{(r)}}{\phi^{(r)T} K \phi^{(r)}} \right\} \qquad (15)$$

The above optimization problems were solved using the genetic search algorithm (GA) and sequential quadratic programming (SQP) techniques. This study concluded significant effect of modes of vibration on the optimum locations of the MR fluid segments. Furthermore, it was realized that treating the pockets experiencing significant shear deformation could maximize the loss factor. For instance, the MR fluid treatments closed to the supports of simply supported and clamped sandwich beams yielded higher modal damping factors corresponding to all the modes.

Snamina (2011) employed the classical plate theory and energy method to find optimal number and locations of the MR segments in the core layer of a sandwich plate to maximize the energy dissipation of the structure. The objective function was considered as

$$\text{Maximize}\, P = \frac{1}{2}\omega A^2 \text{Im}(G)\int_V (\gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2)dV \quad (16)$$

where P is the average power dissipated from the plate; ω and A denote the frequency and amplitude of excitation, respectively; $\text{Im}(G)$ is the imaginary part of the complex shear modulus of the MR fluid; and γ represents shear strain of the MR fluid. This study revealed greater number of optimal MR fluid treatments for the higher modes of vibration. Mohammadi and Sedaghati (2012c) formulated an optimization problem to maximize damping corresponding to the first two modes of a sandwich panel partially treated with ER fluid. They considered locations of the ER segments, thickness ratios of the face, and core layers and electric field intensity as the design variables and employed GA and SQP techniques to find optimal design. Furthermore, a constraint was defined so that the total mass of the sandwich structure should not increase the mass of the base layer more than 50%. The optimization results showed that the maximum electric field intensity would yield the highest loss factor, irrespective of the boundary condition. Ni et al. (2010) fabricated a sandwich beam with MRE as the core to suppress micro vibration of equipment under stochastic support vibration. They formulated an optimization problem and considered velocity response spectra and the root mean square (RMS) velocity responses of the structure as the objective functions, while the applied magnetic field was assumed as the design variable. The results accentuated significant effect of optimal MRE on reducing RMS velocity responses of the structure.

Vibration control of MR/ER sandwich structures

Application of smart fluids in the structures enables to develop semi-active controller to attenuate unwanted vibration in a wide range of frequency. This is due to tunable stiffness and damping characteristics of MR/ER-based sandwich structures in response to the applied field. Semi-active controllers pose the simplicity and reliability of the passive controllers as well as effectiveness and adaptability of the active ones. Kim et al. (1992) compared performance of semi-active controller with that of passive system, under critical and maximum damping, on a cantilever ER sandwich beam and reported superior functionality of former controller. Furthermore, their study indicated that semi-active system was not sensitive to spillover problem comparing with fully active system. Active controllers require force and torque inputs from actuator to suppress vibration; thus, application of these controllers is limited due to high cost and power requirements. Moreover, they are prone to instability. Although control study of the MR and ER dampers has been widely investigated, control analysis of the sandwich structures incorporating MR and ER fluids has been addressed only in few studies. Semi-active controllers have been applied to the MR/ER sandwich structures through various control strategies such as ON–OFF control law (Liao et al., 2012; Sapiński and Snamina, 2008), linear quadratic regulator (LQR) (Rajamohan et al., 2011), sliding mode (Allahverdizadeh et al., 2013c; Hasheminejad et al., 2013; Kim et al., 1992), and real-time control (Zhang and Li, 2009). The performance of the controllers was demonstrated by suppressing external disturbances such as sinusoidal signal (Choi et al., 1996; Fukuda et al., 2000; Liao et al., 2012), random signal (Allahverdizadeh et al., 2013c; Choi et al., 1996; Han et al., 1994; Liao et al., 2012), impulse (Allahverdizadeh et al., 2013c; Liao et al., 2012; Rajamohan et al., 2011), and white noise (Rajamohan et al., 2011). Furthermore, some studies investigated the effects of semi-active controllers on free vibration of sandwich structures (Cho et al., 2005; Rajamohan et al., 2011).

Semi-AC of sandwich structures

Vibration control in the adaptive structures is directly connected to control of their modes; therefore, the governing equations of motion of the structures are generally expressed in the modal form using modal coordinates (Allahverdizadeh et al., 2013c; Rajamohan et al., 2011). In view of uniform distributed variable damping properties of the sandwich structures, which are provided by MR/ER fluid core layers, proportional damping assumption can be employed in the vibration analysis of the structures. Employing modal coordinate system yields uncoupled governing equations of motion for the sandwich structure in the following form (Allahverdizadeh et al., 2013c; Rajamohan et al., 2011)

$$\{\ddot{\lambda}_i\} + [2\xi_i\omega_i]\{\dot{\lambda}_i\} + \{\omega_i^2\}\{\lambda_i\} = \{f_i\}, \quad i = 1, 2, \ldots, n \quad (17)$$

where $\{\lambda\}$ and $\{f\}$ denote the modal coordinate and force vectors, respectively. ω_i and ξ_i are the natural frequency and corresponding modal damping ratio for the ith normal mode, respectively. The modal parameters of the MR/ER-based sandwich structures show significant variations in response to applied field; hence, the natural frequencies and modal damping factor of the structures can be represented as a function of controlled magnetic/electric field, u_i

$$\{\ddot{\lambda}_i\} + C(u_i)\{\dot{\lambda}_i\} + K(u_i)\{\lambda_i\} = \{f_i\}, \quad i = 1, 2, \ldots, n \quad (18)$$

where $K(u_i) = [\omega_i^2]$ and $C(u_i) = [2\xi_i\omega_i]$. In order to realize an appropriate controller over the structure, the variations in $K(u_i)$ and $C(u_i)$ with the applied field should be identified. It is widely reported that these two

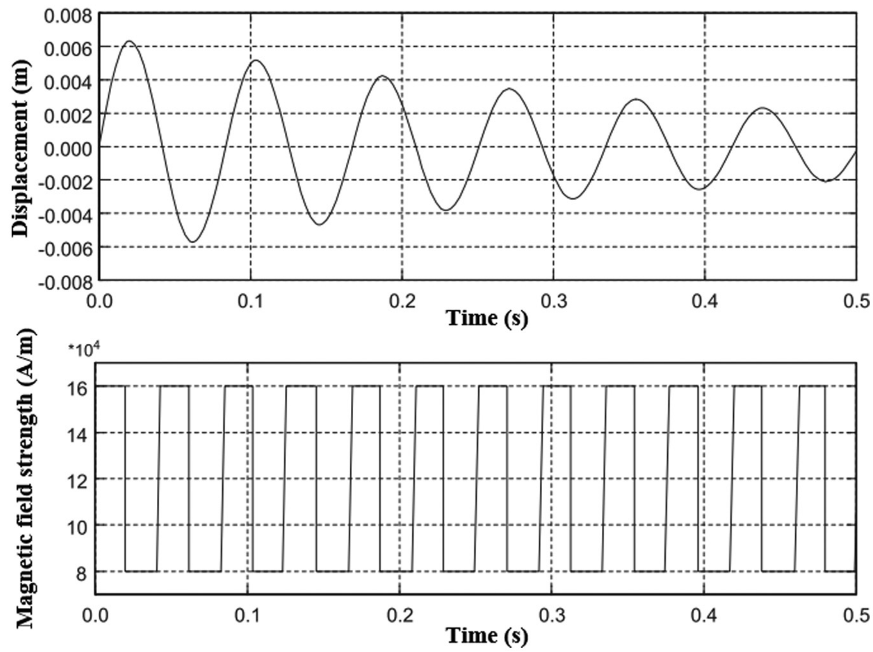


Figure 22. Displacement of the beam end point under magnetic field switching (Sapiński and Snamina, 2008).

parameters vary linearly with respect to the applied field (Allahverdizadeh et al., 2013c; Choi et al., 1993; Kim et al., 1992; Rajamohan et al., 2011). However, the approximation is valid in a limited range of frequency and applied field. The field-dependent equation presented in equation (18) can be used to develop different semi-active controllers such as LQR (Rajamohan et al., 2011) and sliding mode (Allahverdizadeh et al., 2013c) for the sandwich structures.

Sapiński and Snamina (2008) demonstrated the concept of switched stiffness in vibration analysis of a cantilever beam with MR fluid as the core layer. They employed FE model to derive governing equations of motion of the structure and considered free end displacement and velocity of the sandwich beam to design the controller. Their results revealed that switching stiffness to low value, while displacement reached the maximum, dissipated energy from the structure. The control low was also given as

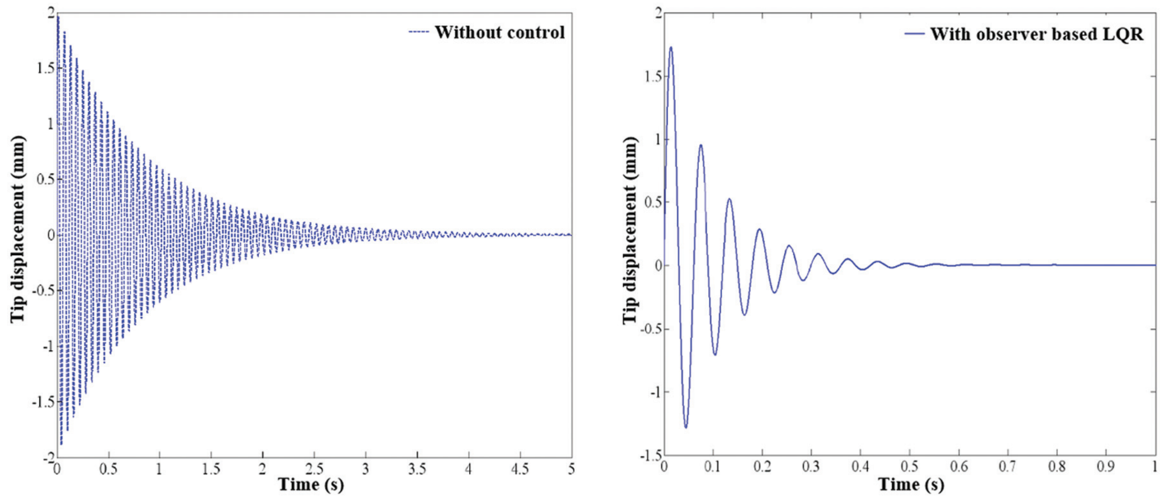
$$\begin{cases} k = k_{\max} & \text{if} \quad w\dot{w} \geq 0 \\ k = k_{\min} & \text{if} \quad w\dot{w} < 0 \end{cases} \tag{19}$$

where w and $\dot{w}$ are the displacement and velocity at the free end of sandwich beam, respectively. Figure 22 illustrates the relation between beam end motion and stiffness switching. The stiffness of the structure is varied under different levels of magnetic flux density. As can be observed in the figure, the magnetic flux density remains at the highest value while displacement increases and switches to the lowest value when the vibration amplitude is maximum. During stiffness switching, the displacement amplitude does not vary; thus, the part of the potential energy is lost from the system. In the next half-cycle, the stiffness is switched to the highest value while the system passes through the equilibrium position. In the equilibrium position, the potential energy of the system is equal to 0; thus, switching the stiffness does not change total energy of the system.

While most of the studies on control of sandwich structures with MR/ER fluids developed controller over fully treated sandwich structures under fully activation of the core layer, there are few studies addressing control synthesis of partially treated sandwich structures or partially activation of the MR/ER fluids. Dyniewicz et al. (2015) proposed a semi-active controller to minimize unwanted vibration of a partially treated MRE beam structure in a particular mode of vibration. Rajamohan et al. (2011) presented full-state observer-based and limited state LQR controller to suppress free and forced vibration of a cantilever beam, comprising two elastic layers and MR fluid as the core layer. The FE models of the fully and partially treated beams were expressed in the state-space form, as

$$\{\dot{x}_s\} = [A]\{x_s\} + [B]\{\gamma\} + \{f\} \quad \text{and} \quad \{y_s\} = [C]\{x_s\} \tag{20}$$

where $[A]$, $[B]$, and $[C\}$ are the coefficient matrices obtained from the governing equations of motion of the structure, $\{\gamma\} = [U]\{x_s\}$, and $[U]$ is the control input matrix. $\{f\}$ is the force vector and $\{x_s\}$ represents the generalized coordinates and their derivatives. An optimal control was synthesized through minimization of a cost function that was proportional to a measure



(a) (b)

Figure 23. Tip deflection responses of the fully treated MR sandwich beam with and without the LQR control and subject to a unit load impulse: (a) time-history: without control and (b) time-history: with observer-based control (Rajamohan et al., 2011).

of the system's response and the desired control inputs, using the LQR approach, such that

$$J = \frac{1}{2}\int_0^\infty \left(\{x_s\}^T[Q]\{x_s\} + \{\gamma\}^T[R]\{\gamma\}\right)dt \qquad (21)$$

where $[Q]$ and $[R]$ are the symmetric semi-definite and positive-definite weighting matrices, respectively. The LQR controller based on the full-state dynamic observer was also formulated to derive the control gain matrix. The dynamic state observer was given by

$$\{\dot{\hat{x}}_s\} = [A]\{\hat{x}_s\} + [B]\{\gamma\} + [L](\{y_s\} - \{\hat{y}_s\}) + \{f\}$$
$$\text{and} \quad \{\hat{y}_s\} = [C]\{\hat{x}_s\} \qquad (22)$$

where $\{\hat{x}_s\}$ is the estimated state vector and $[L]$ is the observer gain evaluated based on LQR control law and determines the convergence of $\{\hat{x}_s\}$ to $\{x_s\}$ and $\{\hat{y}_s\}$ is the output vector evaluated from $\{\hat{x}_s\}$. Application of full-state observer-based LQR control decreased tip displacement as well as settling time of the free vibration, as depicted in Figure 23. The study showed that vibration control of the partially treated beam might be as well as fully treated one if the MR treatments were located near free end of the beam. For instance, the settling time of the fully treated controlled beam was 0.59 s while that of the passive structure was in order of 4.3 s. Interestingly, the settling time of controlled partially treated beam was measured to be 0.53 s, which was lower than that of fully treated beam.

Park et al. (1998) investigated dynamic responses and shape control of a sandwich plate containing ER fluid as the core layer. The host layer was partitioned to provide four cavities. The results illustrated that application of controlled partial ER fluids tuned structural mode shapes and elasto-dynamic properties of the structure, significantly. Cho et al. (2005) reported that application of partially activated ER fluid under sliding mode controller resulted in much less energy consumption than fully activated structure, while almost the same vibration attenuation was acquired. Although it is widely reported that application of controller increases the natural frequencies of the sandwich structures with MR/ER fluids, Rajamohan et al. (2011) suggested no significant shift in the lower three resonant frequencies of the controlled partially treated MR sandwich beam.

AC of MR/ER sandwich structures

While application of semi-active controller on adaptive structures is popular and the semi-active controller is less sensitive to spillover than active controller (Kim et al., 1992), the active controller should be applied; once complete vibration suppression of the adaptive structure is desired (Shaw, 2000). The semi-active controllers can effectively mitigate vibration of the structure in the neighborhood of the resonance frequencies (Choi and Park, 1994). However, non-zero deflection in the structures with semi-active controllers may be observed in different frequencies of excitations. Application of active controllers to mitigate undesired vibration of MR/ER-based sandwich structures has been reported in the literature (Choi et al., 1996; Jianting and Jiesheng, 2003). Choi et al. (1996) involved the actuator characteristics into governing equations of motion of a cantilever sandwich beam containing ER fluid to design an active controller. Fukuda et al. (2000) employed ER fluid core layer and piezoceramic actuator to realize an active controller and suppress

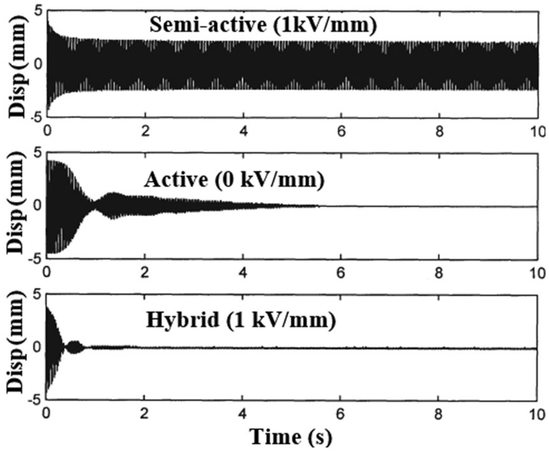


Figure 24. Time response of the second mode excitation, under semi-active, active, and hybrid controllers (Shaw, 2000).

deflection at the free end of the cantilever composite beam. Four types of feedback control strategies including deflection feedback control (DFC) and velocity feedback control (VFC) through or not through a relay element for both of the ER fluid and piezoceramic actuator were considered. The relay element provided a comparison between control effect of the sinusoidal and rectangular waves as the input to the actuators. In other words, four types of control strategies were applied to ER fluid and piezoceramic actuator and the amounts of the control input signals based on DFC and VFC were measured. The results suggested that VFC without a relay element for ER fluid and piezoceramic actuator showed optimal performance in vibration attenuation of the structure, under sinusoidal excitation.

The Lyapunov stability theory has been employed in some studies to apply active controllers for vibration mitigation of sandwich beam structures with ER (Rahn and Joshi, 1998) and MR (Chen and Hansen, 2005; Chen and Tian, 2006) fluids, respectively. Shaw (2000) developed two-stage hybrid controller to attenuate vibration of an ER-based adaptive beam under harmonic disturbances. This controller combined fuzzy logic–based semi-active controller with an active force controller. The semi-active controller tuned the resonant frequencies and improved transient response of the structure, while the active controller eliminated external disturbances and improved steady-state responses. The results highlighted superior performance of hybrid controller over semi-active and active controllers in suppressing undesired vibration. Furthermore, the results reported beating phenomenon in response to AC of the structure which was due to small damping of the structure and negligible deviation of the excitation and natural frequencies of the structure. Figure 24 provides a comparison between semi-active, active, and hybrid controllers in vibration attenuation in the second mode of the structure. Hasheminejad et al. (2013) developed an active controller to control supersonic flutter motion of sandwich plate with ER fluid as the core layer. An arbitrary flow with various yaw angles was applied to the structure mounting on a Winkler–Pasternak elastic foundation.

Conclusion

This article mainly summarizes the studies on pre-yield characterization of MR/ER fluids, dynamic responses of partially and fully treated sandwich beams, plates, shells, and panels containing MR/ER fluids as the core layer, and control strategies applied to the structures. A comprehensive review on different fabrication techniques, experimental methods, mathematical modeling, and methods of solution was presented, and the results based on different assumptions were compared. The effects of different parameters such as applied field, geometry, elastic layer material, boundary conditions, excitation frequency, temperature, and external disturbances on dynamic responses of the structures were thoroughly investigated. Furthermore, the complexities associated with experimental studies and the sources of disagreement between theoretical and experimental studies were addressed. Although most of the studies on sandwich structures employed MR/ER fluids to attenuate undesired vibration and instability of the base layers, applications of these structures as shear mode dampers and vibration absorbers have also been reported. The results suggested that although rheometers have been widely used to characterize MR/ER fluids in terms of applied field and excitation frequency, treating MR/ER sandwich beam structures as SDOF systems ensured pre-yield characterization of the fluids which was due to smaller strain amplitude of the fluids in the sandwich structures compared to the rheometers. Moreover, solid models were more appropriate to identify pre-yield characteristics of the MR/ER fluids. It was further noted that application of MR/ER fluids in sandwich structures subject to magnetic/electric field could significantly alter the stiffness and damping properties of the structures under a noticeable shear strain. Although fully treated sandwich structures generally yielded more significant performance compared to partially treated ones, optimal design of the partially treated MR/ER-based sandwich structures, in some cases, could provide superior damping properties compared with the fully treated ones, while having less weight. The performance of the partially and fully treated structures, however, could be enhanced by developing appropriate semi-active or active controllers over the structures. The results highlighted that semi-active controllers could suppress resonant deflection of the structures while active controllers ensured complete vibration suppression of the adaptive structures in a wide range of frequency.

Declaration of conflicting interests

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References

Aguib S, Nour A, Zahloul H, et al. (2014) Dynamic behavior analysis of a magnetorheological elastomer sandwich plate. *International Journal of Mechanical Sciences* 87: 118–136.

Allahverdizadeh A, Eshraghi I, Mahjoob MJ, et al. (2014) Nonlinear vibration analysis of FGER sandwich beams. *International Journal of Mechanical Sciences* 78: 167–176.

Allahverdizadeh A, Mahjoob MJ, Eshraghi I, et al. (2012) Effects of electrorheological fluid core and functionally graded layers on the vibration behavior of a rotating composite beam. *Meccanica* 47(8): 1945–1960.

Allahverdizadeh A, Mahjoob MJ, Eshraghi I, et al. (2013a) On the vibration behavior of functionally graded electrorheological sandwich beams. *International Journal of Mechanical Sciences* 70: 130–139.

Allahverdizadeh A, Mahjoob MJ, Maleki M, et al. (2013b) Structural modeling, vibration analysis and optimal viscoelastic layer characterization of adaptive sandwich beams with electrorheological fluid core. *Mechanics Research Communications* 51: 15–22.

Allahverdizadeh A, Mahjoob MJ, Nasrollahzadeh N, et al. (2013c) Optimal parameters estimation and vibration control of a viscoelastic adaptive sandwich beam incorporating an electrorheological fluid layer. *Journal of Vibration and Control*. Epub ahead of print 29 May. DOI: 10.1177/1077546313483159.

ASTM E756-05:2002 (2002) Standard test method for measuring vibration-damping properties of materials.

Berg CD, Evans LF and Kermode PR (1996) Composite structure analysis of a hollow cantilever beam filled with electro-rheological fluid. *Journal of Intelligent Material Systems and Structures* 7(5): 495–502.

Bishay PL, Tawfik M and Negm HM (2010) Experimental and finite element models of an adaptive magnetorheological sandwich beam. In: *Proceedings of the 17th international congress on sound & vibration*, Cairo, 18–22 July.

Carlson JD, Coulter JP and Duclos TG (1990) Electrorheological fluid composite structures. US Patent 4923057.

Chen L and Hansen CH (2005) *Active Vibration Control of Magnetorheological Sandwich Beam*. Busselton, WA, Australia: Australian Acoustical Society.

Chen L and Tian J (2006) Distributed magnetorheological (MR) fluid damper for active structural vibration control. In: *Proceedings of the international symposium on active control of sound and vibration*, Adelaide, SA, Australia, 18–20 September.

Chen Y, Yuan FG and Conrad H (1994) Damping characteristics of laminate composites containing electrorheological (ER) fluids. In: *Proceedings of the 2nd international conference on intelligent material* (ed CA Rogers and GG Wallace), Williamsburg, VA, 5–8 June, pp. 328–339. Lancaster, PA: Technomic Publishing Co, Inc.

Cho KD, Lee I and Han JH (2005) Dynamic characteristics of ER fluid-filled composite plate using multielectrode configuration. *Journal of Intelligent Material Systems and Structures* 16(5): 411–419.

Choi SB (2000) Electric field-dependent vibration characteristics of a plate featuring an electrorheological fluid. *Journal of Sound and Vibration* 234(4): 705–712.

Choi SB and Park YK (1994) Active vibration control of a cantilevered beam containing an electro-rheological fluid. *Journal of Sound and Vibration* 172: 428–432.

Choi SB, Gandhi MV and Thompson BS (1989a) An active vibration tuning methodology for smart flexible structures incorporating electro-rheological fluids: a proof-of-concept investigation. In: *Proceedings of the American control conference*, Pittsburgh, PA, 21–23 June, pp. 694–703. New York: IEEE.

Choi SB, Park YK and Cheong CC (1996) Active vibration control of intelligent composite laminate structures incorporating an electro-rheological fluid. *Journal of Intelligent Material Systems and Structures* 7(4): 411–419.

Choi SB, Park YK and Jung SB (1999) Modal characteristics of a flexible smart plate filled with electrorheological fluids. *Journal of Aircraft* 36(2): 458–464.

Choi SB, Park YK and Kim JD (1993) Vibration characteristics of hollow cantilevered beams containing an electrorheological fluid. *International Journal of Mechanical Sciences* 35(9): 757–768.

Choi SB, Park YK and Suh MS (1994) Elastodynamic characteristics of hollow cantilever beam containing an electrorhoelogical fluid: experimental results. *AIAA Journal* 32(2): 438–440.

Choi SB, Seo JW, Kim JH, et al. (2001) An electrorheological fluid-based plate for noise reduction in a cabin: experimental results. *Journal of Sound and Vibration* 239(1): 178–185.

Choi SB, Thompson BS and Gandhi MV (1989b) An experimental investigation on the active damping characteristics of a class of ultra-advanced intelligent composite materials featuring electro-rheological fluids. In: *Proceedings of the damping '89*, West Palm Beach, FL, 8–10 February, pp. 1–14. Columbus, OH: Air Force Systems Command, Wright-Patterson Air Force Base.

Choi SB, Thompson BS and Gandhi MV (1989c) Smart structures incorporating electro-rheological fluids for vibration-control and active-damping applications: an experimental investigation. In: *Proceedings of the 12th biennial ASME conference on mechanical vibration and noise*, Montreal, QC, Canada, 17–21 September.

Choi WJ, Xiong YP and Shenoi RA (2008) Characterization of magnetorheological elastomer materials for the core of smart sandwich structures. In: *Proceedings of the 8th international conference on sandwich structures*, Porto, 6–8 May, pp. 818–826.

Choi WJ, Xiong YP and Shenoi RA (2010) Vibration characteristics of sandwich beams with steel skins and magnetorheological elastomer cores. *Advances in Structural Engineering* 13(5): 837–847.

Choi Y, Sprecher AF and Conrad H (1990) Vibration characteristics of a composite beam containing an electrorheological fluid. *Journal of Intelligent Material Systems and Structures* 1: 91–104.

Choi Y, Sprecher AF and Conrad H (1992) Response of electrorheological fluid-filled laminate composites to forced vibration. *Journal of Intelligent Material Systems and Structures* 3(1): 17–29.

Claracq J, Sarrazin J and Montfort J (2004) Viscoelastic properties of magnetorhelogical fluids. *Rheologica Acta* 43: 38–49.

Coulter JP and Duclos TG (1990) Applications of electrorheological materials in vibration control. In: *Proceedings of the second international conference on ER fluids* (ed JD Carlson), Lancaster, UK, 11–14 March 1990.

Coulter JP, Don DL, Yalcintas M, et al. (1993a) An experimental investigation of ER material based adaptive plates. In: *Proceedings of the ASME winter annual*, New Orleans, LA, 28 November–3 December.

Coulter JP, Don DL, Yalcintas M, et al. (1993b) Experimental investigation of electro-rheological material based adaptive plates. *Adaptive Structures and Material Systems* 35: 287–296.

Coulter JP, Duclos TG and Acker DN (1989) The usage of electrorheological materials in viscoelastic layer damping applications. In: *Proceedings of the damping '89*, West Palm Beach, FL, 8–10 February.

Coulter JP, Weiss KD and Carlson JD (1993c) Engineering applications of electrorheological materials. *Journal of Intelligent Material Systems and Structures* 4(2): 248–259.

Deng HX and Gong XL (2007) Adaptive tuned vibration absorber based on magnetorheological elastomer. *Journal of Intelligent Material Systems and Structures* 18(12): 1205–1210.

DiTaranto RA (1965) Theory of vibratory bending for elastic and viscoelastic layered finite-length beams. *Journal of Applied Mechanics* 32(4): 881–886.

Don DL (1993) *An investigation of electrorheological material adaptive structures*. Master's Thesis, Lehigh University, Bethlehem, PA.

Don DL and Coulter JP (1995) An analytical and experimental investigation of electrorheological material based adaptive beam structures. *Journal of Intelligent Material Systems and Structures* 6(6): 846–853.

Dwivedy SK and Srinivas M (2008) Parametric instability regions of a soft cored sandwich beam with magnetorheological elastomer and non-conductive skins. In: *Proceedings of the 52nd Congress of the Indian Society of Theoretical and Applied Mechanics*, Bangalore, India, 14–17 December, pp. 117–123. Kharagpur, India: Indian Society of Theoretical and Applied Mechanics.

Dwivedy SK and Srinivas M (2011) Dynamic instability of MRE embedded soft cored sandwich beam with non-conductive skins. *Shock and Vibration* 18: 759–788.

Dwivedy SK, Mahendra N and Sahu KC (2009) Parametric instability regions of a soft and magnetorheological elastomer cored sandwich beam. *Journal of Sound and Vibration* 325: 686–704.

Dyniewicz B, Bajkowski J and Bajer C (2015) Semi-active control of a sandwich beam partially filled with magnetorheological elastomer. *Mechanical Systems and Signal Processing* 60–61: 695–705.

Ehrgott R and Masri S (1992) Experimental studies of electrorheological materials for application in adaptive structures. In: *Proceedings of the 3rd international conference on adaptive structures*, San Diego, CA, 9–11 November, pp. 161–175. Lancaster, PA: Technomic Publishing Co, Inc.

Eshaghi M, Rakheja S and Sedaghati R (2015a) An accurate technique for pre-yield characterization of MR fluids. *Smart Materials and Structures* 24: 65018–65031.

Eshaghi M, Sedaghati R and Rakheja S (2015b) Dynamic characterization of partially treated MR based sandwich plate: finite element modeling and experimental study. In: *Proceedings of the ASME 2014 smart materials, adaptive structures and intelligent systems*, Newport, RI, 8–10 September.

Eshaghi M, Sedaghati R and Rakheja S (2015c) The effect of magneto-rheological fluid on vibration suppression capability of adaptive sandwich plates: experimental and finite element analysis. *Journal of Intelligent Material Systems and Structures* 26: 1920–1935.

Fukuda T, Takawa T and Nakashima K (2000) Optimum vibration control of CFRP sandwich beam using electrorheological fluids and piezoceramic actuators. *Smart Materials and Structures* 9(1): 121–125.

Gamota DR and Filisko FE (1991) Dynamic mechanical studies of electrorheological materials: moderate frequencies. *Journal of Rheology* 35(3): 399–425.

Gandhi F and Bullough WA (2005) On the phenomenological modeling of electrorheological and magnetorheological fluid pre-yield behavior. *Journal of Intelligent Material Systems and Structures* 16: 237–248.

Gandhi MV and Thompson BS (1990) Dynamically-tunable smart composites featuring electro-rheological fluids. *International Society for Optics and Photonics* 1170: 294–304.

Gandhi MV, Thompson BS and Choi SB (1988) Ultra-advanced composite materials incorporating electrorheological fluids. In: *Proceedings of the 4th Japan–US conference on composite materials*, Washington, DC, 27–29 June, pp. 875–884. Lancaster, PA: Technomic Publishing Co, Inc.

Gandhi MV, Thompson BS and Choi SB (1989) A new generation of innovative ultra-advanced intelligent composite materials featuring electro-rheological fluids: an experimental investigation. *Journal of Composite Materials* 23(12): 1232–1255.

Genc S and Phule PP (2002) Rheological properties of magnetorheological fluids. *Smart Materials and Structures* 11: 140–146.

Ginder JM, Davis LC and Elie LD (1995) Rheology of magnetorheological fluids: Models and measurements. In: *Proceedings of the 5th international conference on ER fluids and MR suspensions*, Sheffield, 10–14 July, pp. 504–514. Singapore: World Scientific.

Ginder JM, Schlotter WF and Nichols ME (2001) Magnetorheological elastomers in tunable vibration absorbers. In: *Proceedings of the SPIE's 8th annual international symposium on smart structures and materials*, Newport Beach, CA, 4 March, pp. 103–110. Bellingham, WA: International Society for Optics and Photonics (SPIE).

Haiqing G and King LM (1997) Vibration characteristics of sandwich beams partially and fully treated with electrorheological fluid. *Journal of Intelligent Material Systems and Structures* 8(5): 401–413.

Haiqing G, King LM and Cher TB (1993) Influence of a locally applied electro-rheological fluid layer on vibration of a simple cantilever beam. *Journal of Intelligent Material Systems and Structures* 4(3): 379–384.

Han L, Voloshin AS, Yalcintas M, et al. (1994) Electrorheological adaptive structures with embedded sensing and control. In: *Proceedings of the North American conference on smart structures and materials*, Orlando, FL, 13 February, pp. 2–12. Bellingham, WA: International Society for Optics and Photonics (SPIE).

Harland NR, Mace BR and Jones RW (2001) Adaptive-passive control of vibration transmission in beams using electro/magnetorheological fluid filled inserts. *Control Systems Technology* 9(2): 209–220.

Hasheminejad SM and Maleki M (2009) Free vibration and forced harmonic response of an electrorheological fluid-filled sandwich plate. *Smart Materials and Structures* 18(5): 1–16.

Hasheminejad SM and Motaaleghi MA (2014) Supersonic flutter control of an electrorheological fluid-based smart circular cylindrical shell. *International Journal of Structural Stability and Dynamics* 14(2): 1350064–1350088.

Hasheminejad SM and Shabanimotlagh M (2010) Magnetic-field-dependent sound transmission properties of magnetorheological elastomer-based adaptive panels. *Smart Materials and Structures* 19(3): 035006.

Hasheminejad SM, Nezami M and Panah MA (2013) Flutter suppression of an elastically supported plate with electrorheological fluid core under yawed supersonic flows. *International Journal of Structural Stability and Dynamics* 13(1): 1250073–1250097.

Hirunyapruk C, Brennan MJ, Mace BR, et al. (2010) A tunable magneto-rheological fluid-filled beam-like vibration absorber. *Smart Materials and Structures* 19: 055020–055030.

Hosseini-Hashemi SH, Eshaghi M and Karimi M (2010a) Closed-form solution for free vibration of piezoelectric coupled annular plates using Levinson plate theory. *Journal of Sound and Vibration* 329: 1390–1408.

Hosseini-Hashemi SH, Eshaghi M and Rokni Damavandi Taher H (2010b) An exact analytical solution for freely vibrating piezoelectric coupled circular/annular thick plates using Reddy plate theory. *Composite Structures* 92: 1333–1351.

Hosseini-Hashemi SH, Khorshidi K, Eshaghi M, et al. (2012) On the effects of coupling between in-plane and out-of-plane vibrating modes of smart functionally graded circular/annular plates. *Applied Mathematical Modelling* 36(3): 1132–1147.

Hu B, Wang D, Xia P, et al. (2006) Investigation on the vibration characteristics of a sandwich beam with smart composites-MRF. *World Journal of Modelling and Simulation* 2(3): 201–206.

Hu G, Guo M, Li W, et al. (2011) Experimental investigation of the vibration characteristics of a magnetorheological elastomer sandwich beam under non-homogeneous small magnetic fields. *Smart Materials and Structures* 20: 127001–127008.

Huang SC, Inman DJ and Austin EM (1996) Some design considerations for active and passive constrained layer damping treatments. *Smart Materials and Structures* 5(3): 301–313.

Jianting R and Jiesheng J (2003) Active vibration control of an electrorheological sandwich beam. *Mechanics of Electromagnetic Solids* 3: 163–172.

Joshi SB (2012) Vibration study of magnetorheological fluid filled sandwich beams. *International Journal of Applied Research in Mechanical Engineering* 2(2): 100–104.

Kamath GM and Wereley NM (1997) A nonlinear viscoelastic–plastic model for electrorheological fluids. *Smart Materials and Structures* 6: 351–359.

Kang YK, Kim J and Choi SB (2001) Passive and active damping characteristics of smart electro-rheological composite beams. *Smart Materials and Structures* 10(4): 724–729.

Kciuk M and Turczyn R (2006) Properties and application of magnetorheological fluids. *Journal of Achievements in Materials and Manufacturing Engineering* 18(1): 127–130.

Kim Y, Wang K and Lee H (1992) Feedback control of ER-fluid-based structures for vibration suppression. *Smart Materials and Structures* 1: 139.

Kordonsky W, Mastepuro A, Demchuk S, et al. (1994) Electrorheological fluid-based composite materials properties for vibration control in distributed systems. In: *Proceedings of the international conference on intelligent materials (ICIM'94)*, Williamsburg, VA, 5–8 June, pp. 743–750. Lancaster, PA: Technomic Publishing Co, Inc.

Korobko E, Mikhasev G, Novikova Z, et al. (2012) On damping vibrations of three-layered beam containing magnetorheological elastomer. *Journal of Intelligent Material Systems and Structures* 23: 1019–1023.

Lara-Prieto V, Parkin R, Jackson M, et al. (2010) Vibration characteristics of MR cantilever sandwich beams: experimental study. *Smart Materials and Structures* 19: 15005–15014.

Lee C (1995) Finite element formulation of a sandwich beam with embedded electro-rheological fluids. *Journal of Intelligent Material Systems and Structures* 6(5): 718–728.

Lee CY (1992) Dynamic characteristics of a smart beam featuring embedded electro-rheological fluids. In: *Proceedings of the 9th international conference on mechanical engineering, CSME*, Kaohsiung, Taiwan, 11–14 February 1992, pp. 441–448.

Lee CY and Cheng CC (1998) Dynamic characteristics of sandwich beam with embedded electro-rheological fluid. *Journal of Intelligent Material Systems and Structures* 9(1): 60–68.

Lee CY and Cheng CC (2000) Complex moduli of electrorheological material under oscillatory shear. *International Journal of Mechanical Sciences* 42(3): 561–573.

Leng J, Liu Y and Du S (1997) Dynamic characteristics of a beam specimen featuring electrorheological fluids. *Experimental Mechanics* 37: 1–4.

Li W and Zhang X (2008) Research and applications of MR elastomers. *Recent Patents on Mechanical Engineering* 1: 161–166.

Li WH, Chen G and Yeo SH (1999) Viscoelastic properties of MR fluids. *Smart Materials and Structures* 8: 460–468.

Li WH, Zhang PQ and Gong XL *Kosasih* (2005) Linear viscoelasticity of MR fluids: dependence on magnetic fields. *International Journal of Modern Physics B* 19(07n09): 1198–1204.

Li WH, Zhou Y and Tian TF (2010) Viscoelastic properties of MR elastomers under harmonic loading. *Rheologica Acta* 49(7): 733–740.

Liao GJ, Gong XL, Xuan SH, et al. (2012) Development of a real-time tunable stiffness and damping vibration isolator

based on magnetorheological elastomer. *Journal of Intelligent Material Systems and Structures* 23(1): 25–33.
Librescu L and Hause T (2000) Recent developments in the modeling and behavior of advanced sandwich constructions: a survey. *Composite Structures* 48(1): 1–17.
Lu H and Meng G (2006) An experimental and analytical investigation of the dynamic characteristics of a flexible sandwich plate filled with electrorheological fluid. *The International Journal of Advanced Manufacturing Technology* 28(11–12): 1049–1055.
Lu J and Li Q (2007) Dynamic performance of cantilever mortar beams with embedded electro-rheological fluids. In: *2005 IMAC-XXIII: conference & exposition on structural dynamics*, Beijing, China, 8–11 May 2005.
Mahjoob MJ, Martin HR and Ismail F (1993) Distributed vibration control using electrorheological fluids. In: *Proceedings of the international modal analysis conference (IMAC)*, Kissimmee, FL, 1–4 February, pp. 68–774. Bellingham, WA: International Society for Optics and Photonics (SPIE).
Mead DJ and Markus S (1969) The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions. *Journal of Sound and Vibration* 10(2): 163–175.
Mead DJ and Markus S (1970) Loss factors and resonant frequencies of encastré damped sandwich beams. *Journal of Sound and Vibration* 12(1): 99–112.
Mikhasev GI, Altenbach H and Korchevskaya EA (2014) On the influence of the magnetic field on the eigenmodes of thin laminated cylindrical shells containing magnetorheological elastomer. *Composite Structures* 113: 186–196.
Mikhasev GI, Botogova MG and Korobko EV (2011) Theory of thin adaptive laminated shells based on magnetorheological materials and its application in problems on vibration suppression. In: H Altenbach and VA Eremeyev (eds) *Shell-Like Structures*. Berlin, Heidelberg: Springer, pp. 727–750.
Mohammadi F and Sedaghati R (2012a) Dynamic mechanical properties of an electrorheological fluid under large amplitude oscillatory shear strain. *Journal of Intelligent Materials and Structures* 23(10): 1093–1105.
Mohammadi F and Sedaghati R (2012b) Nonlinear free vibration analysis of sandwich shell structures with a constrained electrorheological fluid layer. *Smart Materials and Structures* 21(7): 1–18.
Mohammadi F and Sedaghati R (2012c) Vibration analysis and design optimization of sandwich cylindrical panels fully and partially treated with electrorheological fluid materials. *Journal of Intelligent Material Systems and Structures* 23(15): 1679–1697.
Mohammadi N, Mahjoob MJ, Kaffashi B, et al. (2010) An experimental evaluation of pre-yield and post-yield rheological models of magnetic field dependent smart materials. *Journal of Mechanical Science and Technology* 24(9): 1829–1837.
Mohanty SC (2013) Dynamic stability of a sandwich beam with an electrorheological fluid core. In: *Proceedings of the Asia-Pacific international congress on engineering and natural sciences (APICENS-2013)*, Bangkok, Thailand, 16–18 April.
Murata H (2012) Polymerization, ch. 17 (ed AS Gomes). Rijeka: InTech.
Narayana VG and Ganesan N (2007) Critical comparison of viscoelastic damping and electrorheological fluid core damping in composite sandwich skew plates. *Composite Structures* 80(2): 221–233.
Nashif AD, Jones DIG and Henderson JP (1985) *Vibration Damping*. New York: John Wiley & Sons.
Nayak B, Dwivedy SK and Murthy KSRK (2010) Free vibration control of MRE embedded viscoelastic cored sandwich beam with time varying magnetic field. In: *Proceedings of the ASME 10th biennial conference on engineering systems design and analysis*, Istanbul, Turkey, 12–14 July.
Nayak B, Dwivedy SK and Murthy KSRK (2011) Dynamic analysis of magnetorheological elastomer-based sandwich beam with conductive skins under various boundary conditions. *Journal of Sound and Vibration* 330: 1837–1859.
Nayak B, Dwivedy SK and Murthy KSRK (2012a) Multi-frequency excitation of magnetorheological elastomer-based sandwich beam with conductive skins. *International Journal of Non-Linear Mechanics* 47: 448–460.
Nayak B, Dwivedy SK and Murthy KSRK (2014) Dynamic stability of a rotating sandwich beam with magnetorheological elastomer core. *European Journal of Mechanics: A/Solids* 47: 143–155.
Nayak B, Sastri JBS, Dwivedy SK, et al. (2012b) A comparative study of the classical and higher order theory for free vibration analysis of MRE cored sandwich beam with composite skins using finite element method. In: *Proceedings of the IEEE: international conference on advances in engineering, science and management*, Nagapattinam, India, 30–31 March.
Ni YQ, Ying ZG and Chen ZH (2010) Magneto-rheological elastomer (MRE) based composite structures for micro-vibration control. *Earthquake Engineering and Engineering Vibration* 9(3): 345–356.
Ni YQ, Ying ZG and Chen ZH (2011) Micro-vibration suppression of equipment supported on a floor incorporating magneto-rheological elastomer core. *Journal of Sound and Vibration* 330: 4369–4383.
Oyadiji SO (1996) Applications of electro-rheological fluids for constrained layer damping treatment of structures. *Journal of Intelligent Material Systems and Structures* 7(5): 541–549.
Panah MA and Hasheminejad SM (2010) Vibration suppression of an elastic plate by use of an electrorheological patch and constraining layer. *The International Journal of Structural Changes in Solids* 2(2): 65–76.
Park YK, Choi SB and Cheong CC (1994) Dynamic modelling and vibration control of smart structures incorporating electro-rheological fluids. In: *Proceedings of the fifth international conference on adaptive structures*, Sendai, Japan, 3–6 September, pp. 228–237.
Park YK, Choi SB and Jung SB (1998) Shape control of an electrorheological fluid-based smart plate. In: *Proceedings of the 5th annual international symposium on smart structures and materials*, San Diego, CA, 1 March, pp. 824–835. Bellingham, WA: International Society for Optics and Photonics (SPIE).
Phani SA and Venkatraman K (2003) Vibration control of sandwich beams using electro-rheological fluids. *Mechanical Systems and Signal Processing* 17(5): 1083–1095.

Phani SA and Venkatraman K (2005) Damping characteristics of electro-rheological fluid sandwich beams. *Acta Mechanica* 180(1–4): 195–201.
Pranoto T, Nagaya K and Hosoda A (2004) Vibration suppression of plate using linear MR fluid passive damper. *Journal of Sound and Vibration* 276(3): 919–932.
Qiu J, Tani J and Hajika T (1999) Damping effect of multi-layer beams with embedded electro-rheological fluid. *Journal of Intelligent Material Systems and Structures* 10(7): 521–529.
Rahiminasab J and Rezaeepazhand J (2013) Aeroelastic stability of smart sandwich plates with electrorheological fluid core and orthotropic faces. *Journal of Intelligent Material Systems and Structures* 24(5): 669–677.
Rahn C and Joshi S (1998) Modeling and control of an electrorheological sandwich beam. *Journal of Vibration and Acoustics* 120: 221–227.
Rajamohan V (2013) Vibration analysis of a rotating axially non-homogeneous magnetorheological fluid sandwich beam. *Transaction of Control and Mechanical Systems* 2(2): 83–91.
Rajamohan V and Natarajan P (2012) Vibration analysis of a rotating magnetorheological fluid sandwich beam. In: *Proceedings of the international conference on advanced research in mechanical engineering*, Tirupati, India, 10 June.
Rajamohan V and Ramamoorthy M (2012) Dynamic characterization of non-homogeneous magnetorheological fluids based multi-layer beam. *Applied Mechanics and Materials* 110: 105–112.
Rajamohan V, Rakheja S and Sedaghati R (2010a) Vibration analysis of partially treated multi-layer beam with magnetorheological fluid. *Journal of Sound and Vibration* 329: 3451–3469.
Rajamohan V, Sedaghati R and Rakheja S (2010b) Optimum design of a multilayer beam partially treated with magnetorheological fluid. *Smart Materials and Structures* 19: 0650002–0650017.
Rajamohan V, Sedaghati R and Rakheja S (2010c) Vibration analysis of a multi-layer beam containing magnetorheological fluid. *Smart Materials and Structures* 19: 015013–015025.
Rajamohan V, Sedaghati R and Rakheja S (2011) Optimal vibration control of beams with total and partial MR-fluid treatments. *Smart Materials and Structures* 20(11): 115016.
Rajamohan V, Sundararaman V and Govindarajan B (2013) Finite element vibration analysis of a magnetorheological fluid sandwich beam. *Procedia Engineering* 64; 603–612.
Ramamoorthy M, Rajamohan V and Jeevanantham AK (2014) Vibration analysis of a partially treated laminated composite magnetorheological fluid sandwich plate. *Journal of Vibration and Control*. Epub ahead of print 14 May. DOI: 10.1177/1077546314532302.
Rezaeepazhand J and Pahlavan L (2008a) Transient response of sandwich beams with electrorheological core. *Journal of Intelligent Material Systems and Structures* 20: 171–179.
Rezaeepazhand J and Pahlavan L (2008b) Transient response of three layer sandwich plate with electrorheological core and orthotropic faces. In: *Proceedings of the 13th European conference on composite materials (ECCM-13)*, Stockholm, 2–5 June.
Ross D, Ungar EE and Kerwin EM (1959) Damping of plate flexural vibrations by means of viscoelastic laminea. In: Ruzicka JE (ed.) *Structural Damping*, vol. 3. New York: ASME, pp. 212–226.
Sapiński B and Snamina J (2008) Vibration control capabilities of a cantilever beam with a magnetorheological fluid. *Mechanics* 27(2): 70–75.
Sapiński B and Snamina J (2009) Modeling of an adaptive beam with MR fluid. *Solid State Phenomena* 147: 831–838.
Sapiński B, Snamina J and Romaszko M (2010) Identification of model parameters of a sandwich beam incorporating magnetorheological fluid. *Vibration in Physical Systems* 24: 349–354.
Sepehrinour M and Nezami M (2012) Dynamic response of an electro-rheological sandwich beam with different elastic layers subjected to simultaneous impact loads. *Applied Mechanics and Materials* 232: 117–121.
Shaw J (2000) Hybrid control of a cantilevered ER sandwich beam for vibration suppression. *Journal of Intelligent Material Systems and Structures* 11(1): 26–31.
Shiang AH and Coulter JP (1994) Controllability and reliability issues related to electrorheological materials adaptive structures. In: *Proceedings of the second international conference on intelligent materials*, Lancaster, PA, 18–21 October, pp. 1117–1130. Lancaster, PA: Technomic Publishing Co, Inc.
Shiang AH and Coulter JP (1996) A comparative study of AC and DC electrorheological material based adaptive structures in small amplitude vibration. *Journal of Intelligent Material Systems and Structures* 7(4): 455–469.
Sims ND and Wereley NM (2003) Modelling of smart fluid dampers. In: *Proceedings of the smart materials and structures: damping and isolation*, vol. 5052, San Diego, CA, 2 March.
Sims ND, Holmes NJ and Stanway R (2004) A unified modelling and model updating procedure for electrorheological and magnetorheological vibration dampers. *Smart Materials and Structures* 13: 100–121.
Snamina J (2011) Optimal location of an active segment of magnetorheological fluid layer in a sandwich plate. In: *Proceedings of the 12th international Carpathian control conference (ICCC)*, Velke Karlovice, 25–28 May, pp. 362–365. New York: IEEE.
Sprecher AF, Carlson JD and Conrad H (1987) Electrorheology at small strains and strain rates of suspensions of silica particles in silicone oil. *Material Science and Engineering* 95: 187–197.
Sprecher AF, Choi Y and Conrad H (1991) Mechanical behavior of ER fluid-filled composites in forced oscillation. In: *Proceedings of the US–Japan conference on adaptive structures, session E*, Lancaster, PA, 24–27 November, pp. 560–579.
Stanway R, Sproston JL and El-Wahed AK (1996) Applications of electro-rheological fluids in vibration control: a survey. *Smart Materials and Structures* 5(4): 464–482.
Stevens NG, Sproston JL and Stanway R (1987) On the mechanical properties of electro-rheological fluids. *Journal of Applied Mechanics* 54(2): 456–458.
Sun Q, Zhou JX and Zhang L (2003) An adaptive beam model and dynamic characteristics of magnetorheological materials. *Journal of Sound and Vibration* 261: 456–481.

Tabassian R and Rezaeepazhand J (2011) Stability of smart sandwich beams with cross-ply faces and electrorheological core subjected to axial loads. *Journal of Reinforced Plastics and Composites* 31(1): 55–64.

Tabassian R and Rezaeepazhand J (2013) Dynamic stability of smart sandwich beams with electro-rheological core resting on elastic foundation. *Journal of Sandwich Structures and Materials* 15(1): 25–44.

Tang X and Conrad H (1996) Quasistatic measurements on a magnetorheological fluid. *Journal of Rheology* 40(6): 1167–1178.

Tylikowski A (2000) Dynamic stability of electrorheological fluid-filled laminate. *Journal of Theoretical and Applied Mechanics* 38(2): 417–428.

Tylikowski A (2002) Analytical modelling of electrorheological material based adaptive shells. *Journal of Theoretical and Applied Mechanics* 40: 761–773.

Vaičaitis R, Liu S and Jotautienė E (2007) Vibrations of a double wall plane adaptive to electrorheological materials. *Journal of Vibroengineering* 9(3): 26–30.

Vaičaitis R, Liu S and Jotautienė E (2008) Nonlinear random vibrations of a sandwich beam adaptive to electrorheological materials. *Mechanika* 3(71): 38–44.

Wang DH and Liao WH (2011) Magnetorheological fluid dampers: a review of parametric modelling. *Smart Materials and Structures* 20: 23001–23035.

Wang Y and Zheng G (2013) Vibration power flow analysis of a submerged constrained layer damping cylindrical shell. *Journal of Vibration and Acoustics* 136(1): 011005–011019.

Wei K, Bai Q, Meng G, et al. (2011) Vibration characteristics of electrorheological elastomer sandwich beams. *Smart Materials and Structures* 20(5): 055012.

Wei K, Meng G, Lu H, et al. (2005) Dynamic analysis of rotating electrorheological composite beams. *International Journal of Modern Physics B* 19(7–9): 1236–1242.

Wei K, Meng G, Zhang W, et al. (2007) Vibration characteristics of rotating sandwich beams filled with electrorheological fluids. *Journal of Intelligent Material Systems and Structures* 18: 1165–1173.

Wei K, Meng G, Zhang W, et al. (2008) Experimental investigation on vibration characteristics of sandwich beams with magnetorheological elastomers cores. *Journal of Central South University of Technology* 15: 239–242.

Wei K, Meng G, Zhou S, et al. (2006) Vibration control of variable speed/acceleration rotating beams using smart materials. *Journal of Sound and Vibration* 298(4): 1150–1158.

Weiss KD, Carlson JD and Nixon DA (1994) Viscoelastic properties of magneto- and electro-rheological fluid. *Journal of Intelligent Materials and Structures* 5: 772–775.

Weiss KD, Duclos TG and Carlson JD (1993) High strength magneto- and electro-rheological fluids. SAE technical paper 932451 (pp. 425–430).

Weiss KD, Duclos TG, Chrzan MJ, et al. (1996) Magnetorheological fluid composite structures. US Patent specification 5547049.

Xu Z, Gong X, Liao G, et al. (2010) An active-damping-compensated magnetorheological elastomer adaptive tuned vibration absorber. *Journal of Intelligent Material Systems and Structures* 21: 1039–1047.

Yalcintas M and Coulter JP (1995a) An adaptive beam model with electrorheological material based applications. *Journal of Intelligent Material Systems and Structures* 6: 498–507.

Yalcintas M and Coulter JP (1995b) Analytical modeling of electrorheological material based adaptive beams. *Journal of Intelligent Material Systems and Structures* 6(4): 488–497.

Yalcintas M and Coulter JP (1995c) Electrorheological material based adaptive beams subjected to various boundary conditions. *Journal of Intelligent Material Systems and Structures* 6(5): 700–717.

Yalcintas M and Coulter JP (1998) Electrorheological material based non-homogeneous adaptive beams. *Smart Materials and Structures* 7(1): 128–143.

Yalcintas M and Dai H (1998) Performance comparison of magnetorheological and electrorheological materials in adaptive structural applications. *Adaptive Structures and Material Systems* 8: 49–61.

Yalcintas M and Dai H (1999) Magnetorheological and electrorheological materials in adaptive structures and their performance comparison. *Smart Materials and Structures* 8: 560–573.

Yalcintas M and Dai H (2004) Vibration suppression capabilities of magnetorheological materials based adaptive structures. *Smart Materials and Structures* 13: 1–11.

Yalcintas M, Coulter JP and Don DL (1995) Structural modeling and optimal control of electrorheological material based adaptive beams. *Smart Materials and Structures* 4(3): 207.

Yanju L, Hejun D and Dianfu W (2001) ER fluid based on inorganic/polymer blend particles and its adaptive viscoelastic properties. *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 189: 203–210.

Yeh JY (2007a) Vibration analyses of the annular plate with electrorheological fluid damping treatment. *Finite Elements in Analysis and Design* 43(11): 965–974.

Yeh JY (2007b) Vibration control of a sandwich annular plate with an electrorheological fluid core layer. *Smart Materials and Structures* 16(3): 837.

Yeh JY (2010a) Dynamic stability control analysis of sandwich annular plate with electrorheological core treatment. *Journal of Engineering Technology and Education* 7(4): 618–627.

Yeh JY (2010b) Vibration and damping characteristics analysis of a rotating annular plate with electrorheological treatment. *Smart Materials and Structures* 19(8): 1–8.

Yeh JY (2011a) Free vibration analysis of rotating polar orthotropic annular plate with ER damping treatment. *Composites Part B: Engineering* 42(4): 781–788.

Yeh JY (2011b) Vibration and damping analysis of orthotropic cylindrical shells with electrorheological core layer. *Aerospace Science and Technology* 15(4): 293–303.

Yeh JY (2012) Active dynamic instability control analysis of polar orthotropic sandwich annular plate with electrorheological fluid damping treatment. *Journal of Engineering Technology and Education* 9(3): 290–299.

Yeh JY (2013) Vibration analysis of sandwich rectangular plates with magnetorheological elastomer damping treatment. *Smart Materials and Structures* 22(3): 1–8.

Yeh JY and Chen LW (2004) Vibration of a sandwich plate with a constrained layer and electrorheological fluid core. *Composite Structures* 65(2): 251–258.

Yeh JY and Chen LW (2005) Dynamic stability of a sandwich plate with a constraining layer and electrorheological fluid core. *Journal of Sound and Vibration* 285(3): 637–652.

Yeh JY and Chen LW (2006) Dynamic stability analysis of a rectangular orthotropic sandwich plate with an electrorheological fluid core. *Composite Structures* 72(1): 33–41.

Yeh JY and Chen LW (2007) Finite element dynamic analysis of orthotropic sandwich plates with an electrorheological fluid core layer. *Composite Structures* 78(3): 368–376.

Yeh JY, Chen JY, Lin CT, et al. (2009) Damping and vibration analysis of polar orthotropic annular plates with ER treatment. *Journal of Sound and Vibration* 325(1): 1–13.

Yeh JY, Chen LW and Wang CC (2004) Dynamic stability of a sandwich beam with a constrained layer and electrorheological fluid core. *Composite Structures* 64(1): 47–54.

Yeh ZF and Shih YS (2005) Critical load, dynamic characteristics and parametric instability of electrorheological material-based adaptive beams. *Computers & Structures* 83(25): 2162–2174.

Yeh ZF and Shih YS (2006a) Dynamic characteristics and dynamic instability of magnetorheological material-based adaptive beams. *Journal of Composite Materials* 40: 1333–1358.

Yeh ZF and Shih YS (2006b) Dynamic stability of a sandwich beam with magnetorheological core. *Mechanics Based Design of Structures and Machines* 34: 181–200.

Yen WS and Achorn PJ (1991) A study of the dynamic behavior of an electrorheological fluid. *Journal of Rheology* 35: 1375–1384.

Yin HM, Sun LZ and Chen JS (2006) Magneto-elastic modeling of composites containing chain-structured magnetostrictive particles. *Journal of the Mechanics and Physics of Solids* 54(5): 975–1003.

Ying ZG and Ni YQ (2009) Micro-vibration response of a stochastically excited sandwich beam with a magnetorheological elastomer core and mass. *Smart Materials and Structures* 18: 95005–95018.

Ying ZG, Chen H and Ni Y (2012) Magnetorheological visco-elastomer and its application to suppressing microvibration of sandwich plates. In: *Proceedings of the third international conference on smart materials and nanotechnology in engineering*, Shenzhen, China, 11 November. Bellingham, WA: International Society for Optics and Photonics (SPIE).

Ying ZG, Ni YQ and Ye SQ (2014) Stochastic micro-vibration suppression of a sandwich plate using a magneto-rheological visco-elastomer core. *Smart Materials and Structures* 23(2): 1–11.

Zhang X and Li W (2009) Adaptive tuned dynamic vibration absorbers working with MR elastomers. *Smart Structures and Systems* 5(5): 517–529.

Zhou GY and Li JR (2003) Dynamic behavior of a magnetorheological elastomer under uniaxial deformation: I. Experiment. *Smart Materials and Structures* 12: 859–872.

Zhou GY and Wang Q (2005) Magnetorheological elastomer-based smart sandwich beams with nonconductive skins. *Smart Materials and Structures* 14: 1001–1009.

Zhou GY and Wang Q (2006a) Study on the adjustable rigidity of magnetorheological-elastomer-based sandwich beams. *Smart Materials and Structures* 15: 59–74.

Zhou GY and Wang Q (2006b) Use of magnetorheological elastomer in adaptive sandwich beam with conductive skins. Part I: magnetoelastic loads in conductive skins. *International Journal of Solids and Structures* 43: 5386–5402.

Zhou GY and Wang Q (2006c) Use of magnetorheological elastomer in an adaptive sandwich beam with conductive skins. Part II: dynamic properties. *International Journal of Solids and Structures* 43: 5403–5420.

Zhou GY, Lin KC and Wang Q (2006) Finite element studies on field-dependent rigidities of sandwich beams with magnetorheological elastomer cores. *Smart Materials and Structures* 15: 787–791.

Appendix I

Notation

B	magnetic flux density
E	electric field strength
f_{io}	natural frequency ratio at $E = 0$
G_r	shear modulus of rubber
G'	storage modulus
G''	loss modulus
G^*	complex shear modulus
rf_i	natural frequency ratio
γ	shear strain
η	loss factor
η_d	structural loss factor
$\{\lambda\}$	modal coordinate
μ	fluid viscosity
ν	Poisson's ratio
ξ	damping ratio
τ	shear stress
τ_y	yield stress
ϕ	mode shape vector
ω	natural frequency